Learning Optimal Intrusion Responses for IT Infrastructures via Decomposition

Visit to New York University

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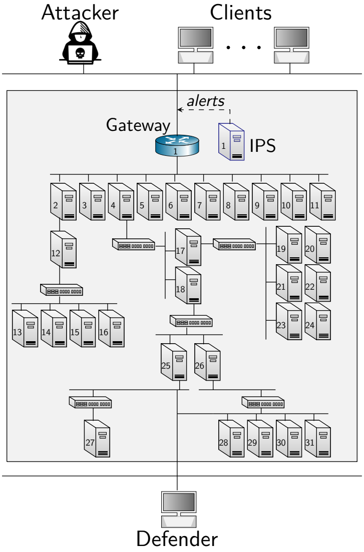
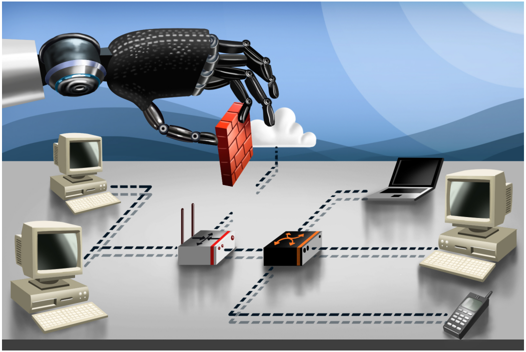
Use Case: Intrusion Response

- A **defender** owns an infrastructure
  - Consists of connected components
  - Components run network services
  - Defender defends the infrastructure by monitoring and active defense
  - Has partial observability
- An **attacker** seeks to intrude on the infrastructure
  - Has a partial view of the infrastructure
  - Wants to compromise specific components
  - Attacks by reconnaissance, exploitation and pivoting



Automated Intrusion Response: Current Landscape



Levels of security automation

No automation.
Manual detection.
Manual prevention.
No alerts.
No automatic responses.
Lack of tools.

1980s

Operator assistance.
Manual detection.
Manual prevention.
Audit logs.
Security tools.

1990s

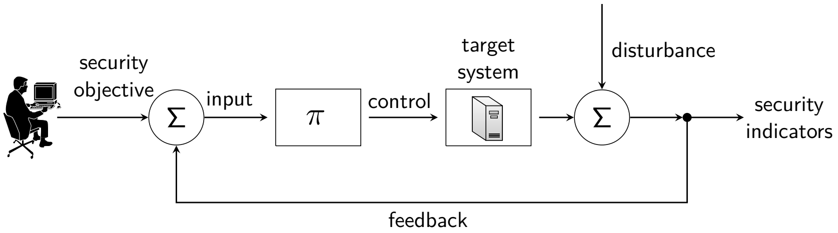
Partial automation.
System has automated functions for detection/prevention but requires manual updating and configuration.
Intrusion detection systems.
Intrusion prevention systems.

2000s-Now

High automation.
System automatically updates itself.
Automated attack detection.
Automated attack mitigation.

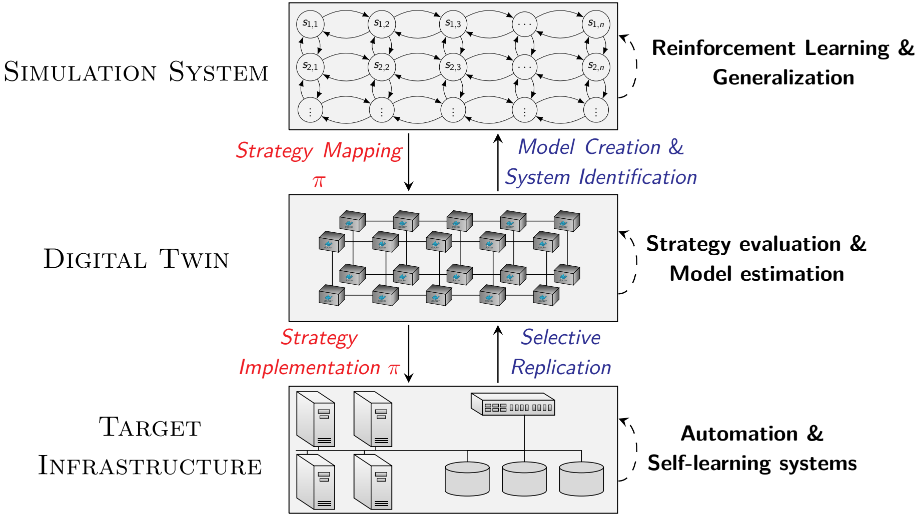
Research

Can we use decision theory and learning-based methods to automatically find effective security strategies?[1]



[1] Kim Hammar and Rolf Stadler. "Finding Effective Security Strategies through Reinforcement Learning and Self-Play". In: *International Conference on Network and Service Management (CNSM 2020)*. Izmir, Turkey, 2020, Kim Hammar and Rolf Stadler. "Learning Intrusion Prevention Policies through Optimal Stopping". In: *International Conference on Network and Service Management (CNSM 2021)*. Izmir, Turkey, 2021, Kim Hammar and Rolf Stadler. "Intrusion Prevention Through Optimal Stopping". In: *IEEE Transactions on Network and Service Management* 19.3 (2022), pp. 2333–2348. DOI: 10.1109/TNSM.2022.3176781, Kim Hammar and Rolf Stadler. *Learning Near-Optimal Intrusion Responses Against Dynamic Attackers*. 2023. DOI: 10.48550/ARXIV.2301.06085. URL: https://arxiv.org/abs/2301.06085.

Our Framework for Automated Network Security



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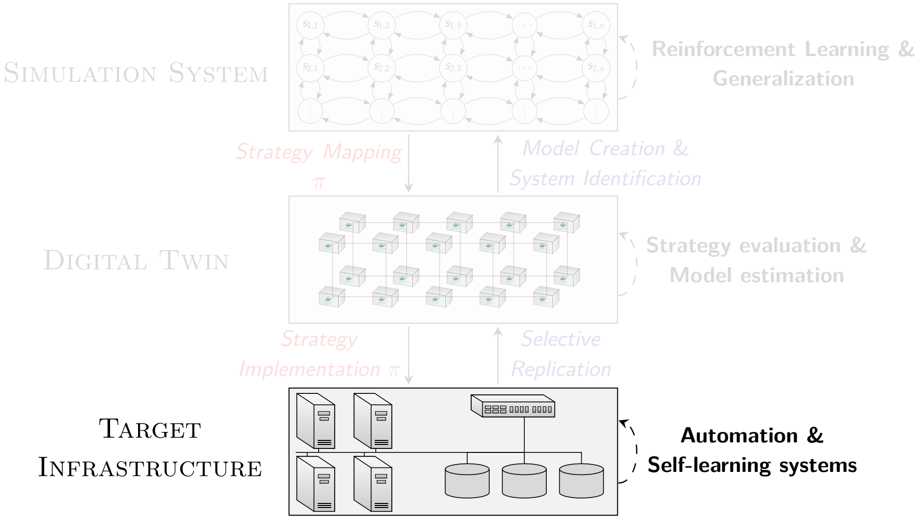

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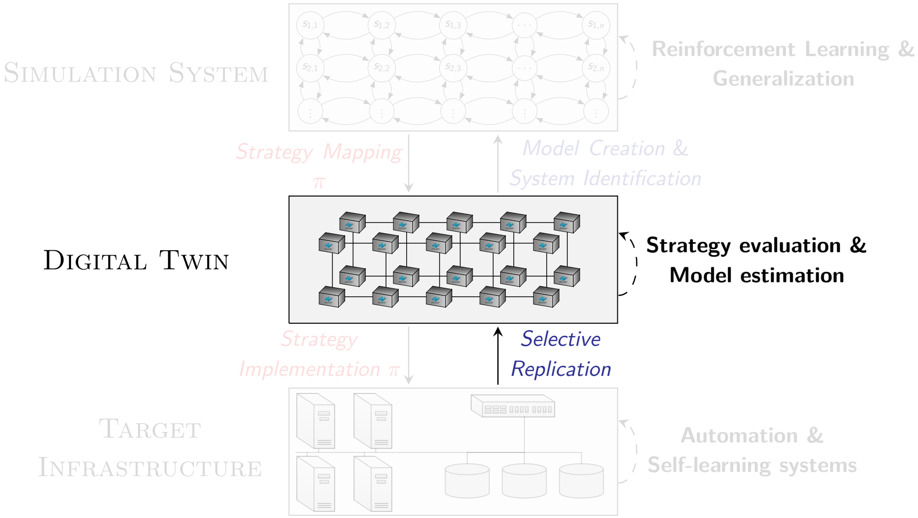

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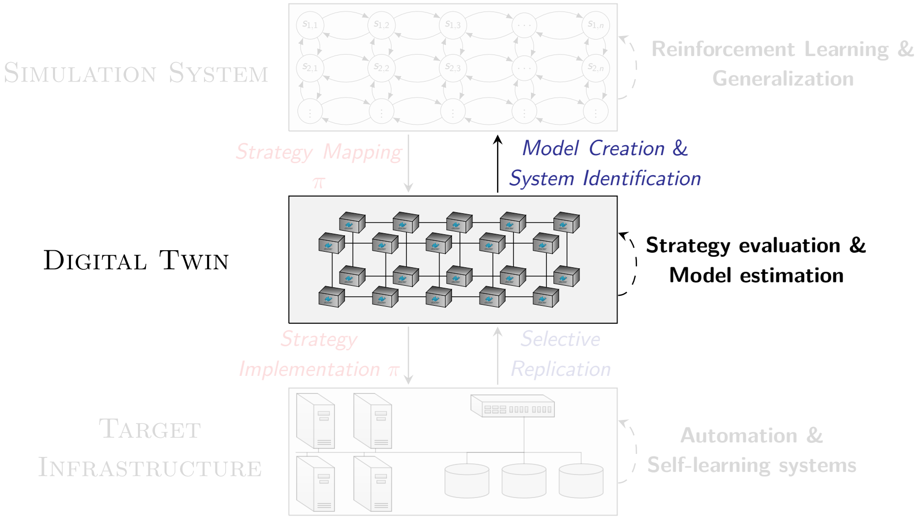

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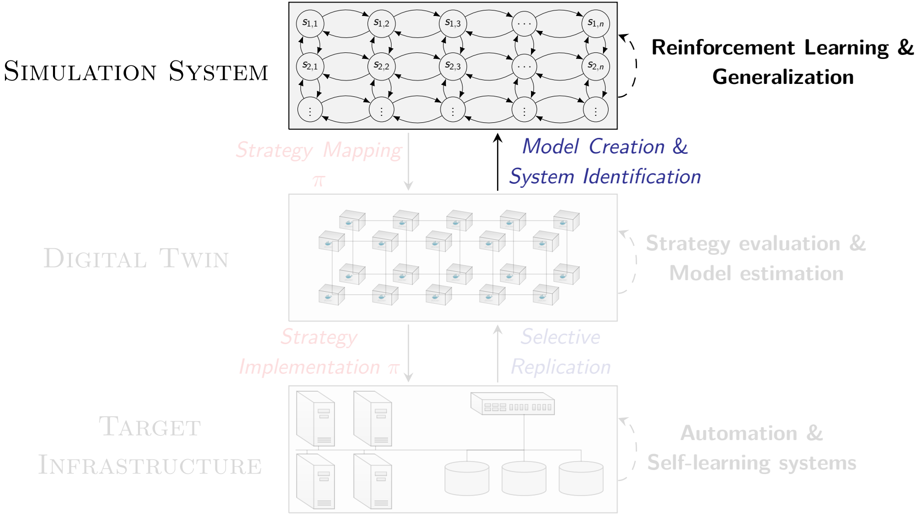

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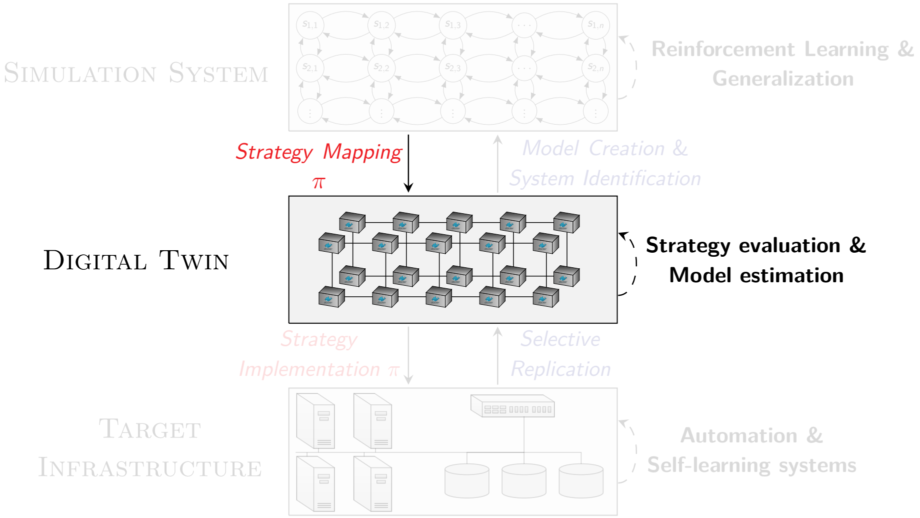

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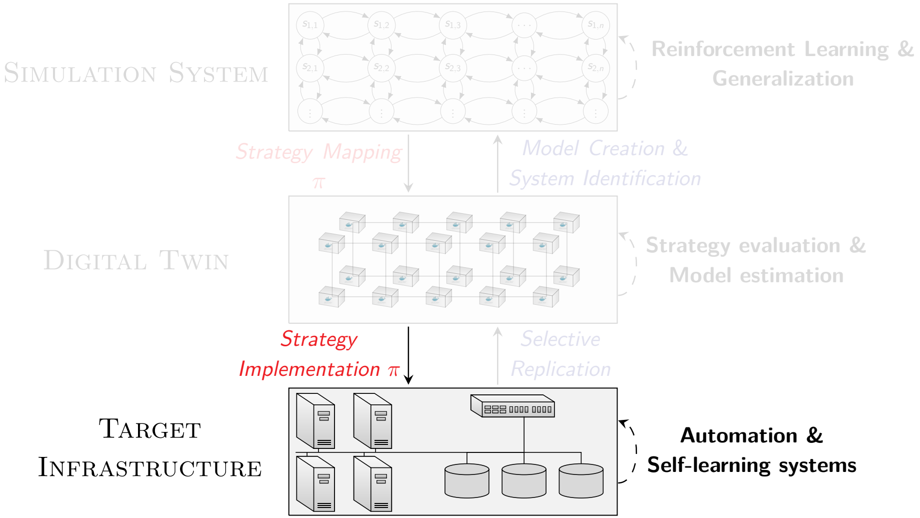

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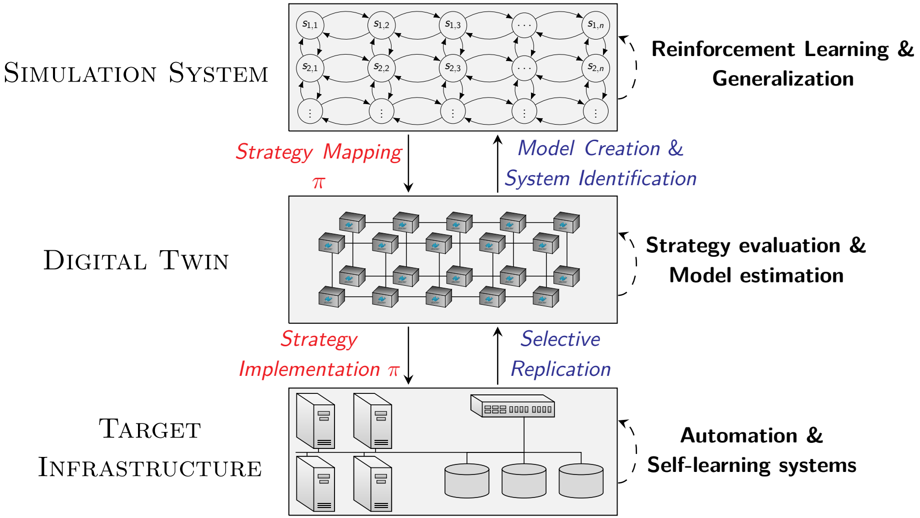
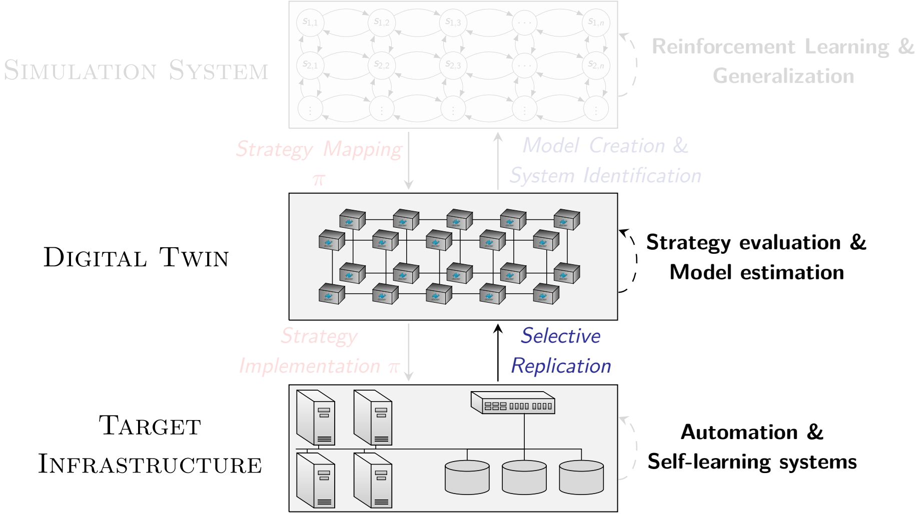
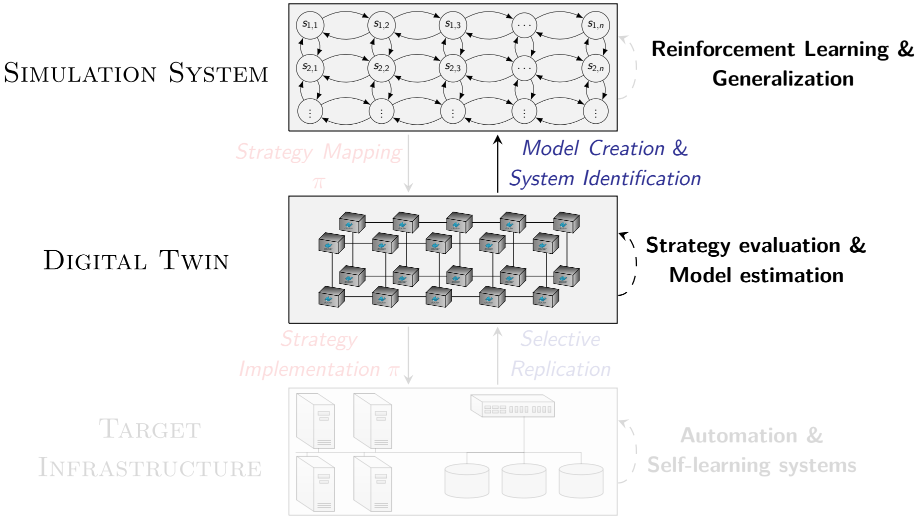


Creating a Digital Twin of the Target Infrastructure

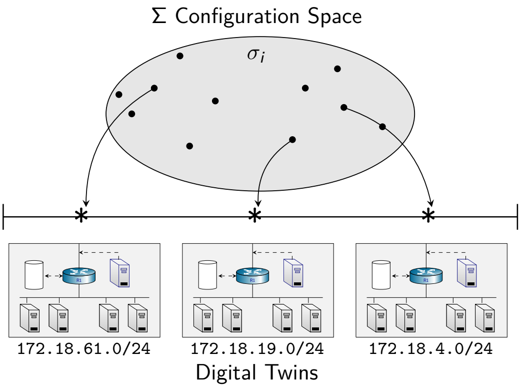


Theoretical Analysis and Learning of Defender Strategies



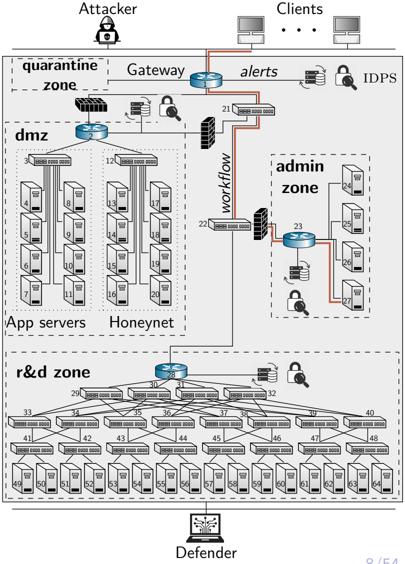
Creating a Digital Twin of the Target Infrastructure

- An infrastructure is defined by its configuration.
- Set of configurations supported by our framework can be seen as a **configuration space**
- The configuration space defines the class of infrastructures for which we can create digital twins.



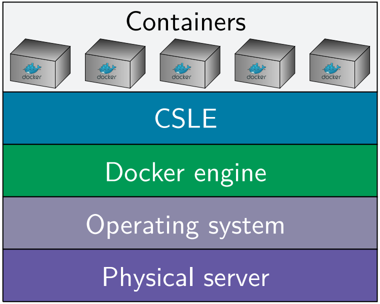
The Target Infrastructure

- 64 **nodes.** 24 OVS switches, 3 gateways. 6 honeypots. 8 application servers. 4 administration servers. 15 compute servers.
- Topology shown to the right
- 11 vulnerabilities (CVE-2010-0426, CVE-2015-3306, CVE-2015-5602, etc.)
- 4 zones: DMZ, R&D ZONE, ADMIN ZONE, QUARANTINE ZONE
- 9 workflows
- Management: 1 SDN controller, 1 Kafka server, 1 elastic server.

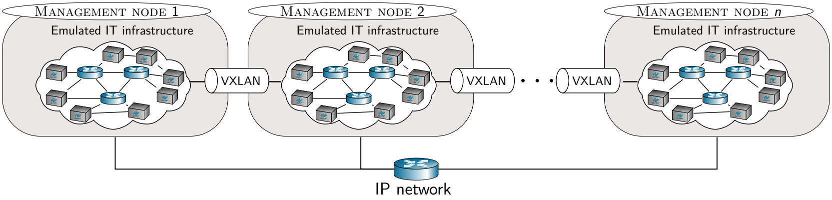


Emulating Physical Components

- We emulate physical components with **Docker containers**
- Focus on linux-based systems
- The containers include everything needed to emulate the host: a runtime system, code, system tools, system libraries, and configurations.
- Examples of containers: IDPS container, client container, attacker container, CVE-2015-1427 container, Open vSwitch containers etc.



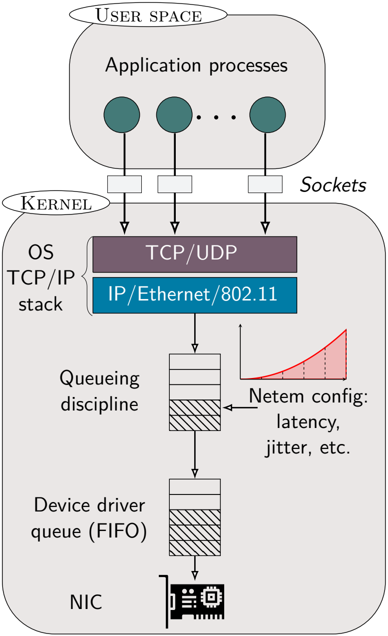
Emulating Network Connectivity



- We emulate network connectivity on the same host using **network namespaces**.
- Connectivity across physical hosts is achieved using **VXLAN tunnels** with Docker swarm.

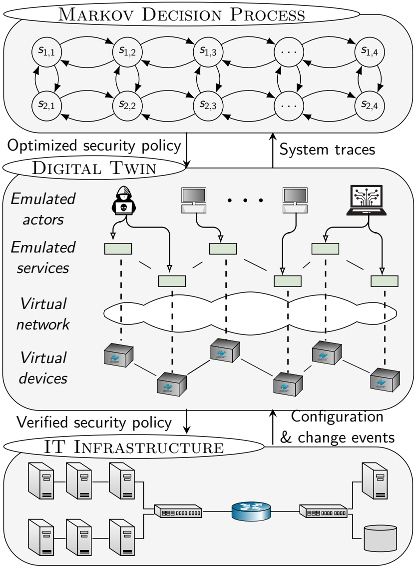
Emulating Network Conditions

- We do traffic shaping using NetEm in the Linux kernel
- Emulate **internal connections** are full-duplex & loss-less with bit capacities of 1000 Mbit/s
- Emulate **external connections** are full-duplex with bit capacities of 100 Mbit/s & 0.1% packet loss in normal operation and random bursts of 1% packet loss

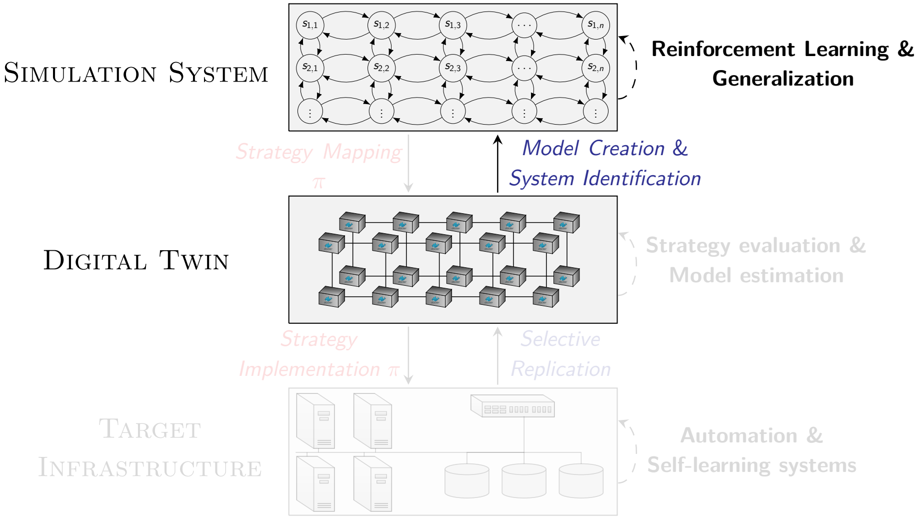


Emulating Actors

- We emulate **client arrivals with Poisson processes**
- We emulate client interactions with load generators
- Attackers are emulated by automated programs that select actions from a pre-defined set
- Defender actions are emulated through a **custom gRPC API**.



System Identification



Outline

- **Use Case & Digital Twin**
  - Use case: intrusion response
  - Digital twin for data collection & evaluation
- **System Model**
  - Discrete-time Markovian dynamical system
  - Partially observed stochastic game
- **System Decomposition**
  - Additive subgames on the workflow-level
  - Optimal substructure on component-level
- **Learning Near-Optimal Intrusion Responses**
  - Scalable learning through decomposition
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  - Efficient equilibrium approximation
- **Conclusions & Future Work**

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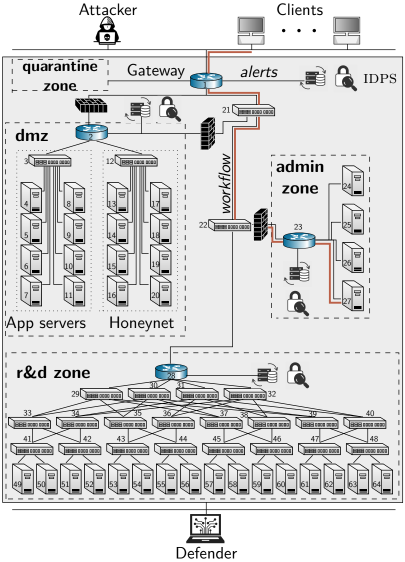
System Model

- $\mathcal{G} = \langle \{\mathrm{gw}\} \cup \mathcal{V}, \mathcal{E} \rangle$: directed graph representing the virtual infrastructure
- $\mathcal{V}$: finite set of virtual components.
- $\mathcal{E}$: finite set of component dependencies.
- $\mathcal{Z}$: finite set of zones.



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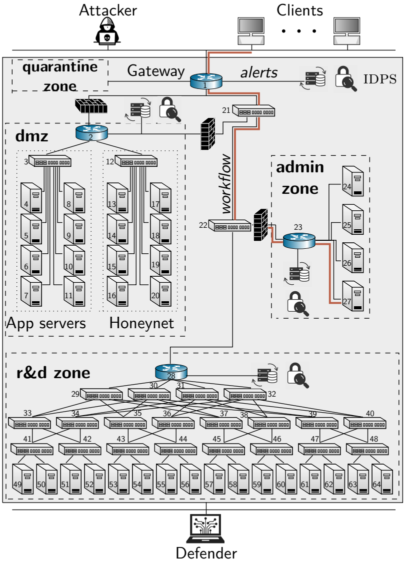

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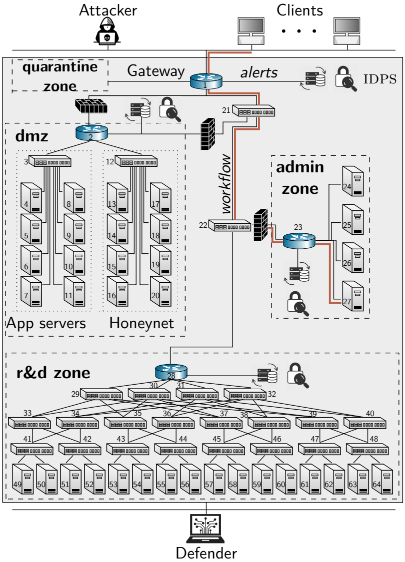


State Model

- Each $i \in \mathcal{V}$ has a state

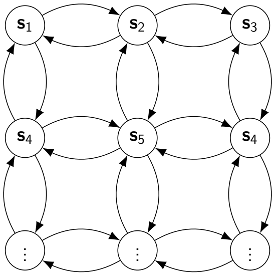
$$\mathbf{v}_{t,i} = (\underbrace{v_{t,i}^{(Z)}}_{\text{D}}, \underbrace{v_{t,i}^{(I)}, v_{t,i}^{(R)}}_{\text{A}})$$

- System state $\mathbf{s}_t = (\mathbf{v}_{t,i})_{i \in \mathcal{V}} \sim \mathbf{S}_t$.

- Markovian time-homogeneous dynamics:

$$\mathbf{s}_{t+1} \sim f(\cdot \mid \mathbf{S}_t, \mathbf{A}_t)$$

$\mathbf{A}_t = (\mathbf{A}_t^{(A)}, \mathbf{A}_t^{(D)})$ are the actions.



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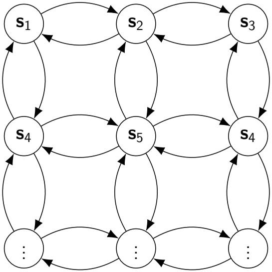

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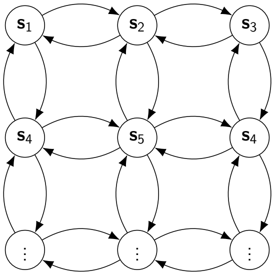

Workflow Model

- Services are connected into **workflows** $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{|\mathcal{W}|}\}$.

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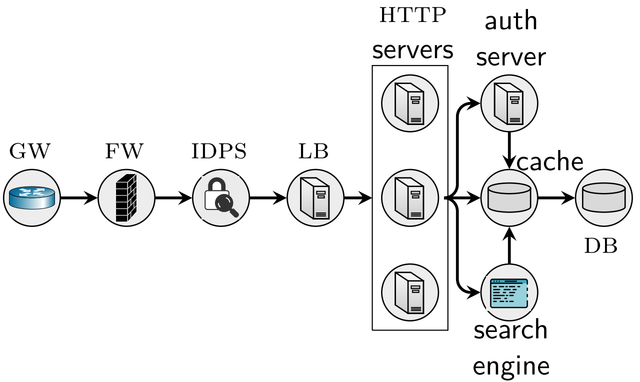
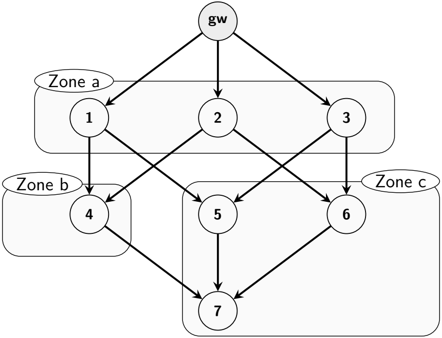


Dependency graph of an example workflow representing a web application; GW, FW, IDPS, LB, and DB are acronyms for gateway, firewall, intrusion detection and prevention system, load balancer, and database, respectively.

Workflow Model

- Services are connected into **workflows** $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{|\mathcal{W}|}\}$.
- Each $\mathbf{w} \in \mathcal{W}$ is realized as a directed acyclic subgraph (DAG) $\mathcal{G}_\mathbf{w} = \langle \{\mathrm{gw}\} \cup \mathcal{V}_\mathbf{w}, \mathcal{E}_\mathbf{w} \rangle$ of $\mathcal{G}$
- $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{|\mathcal{W}|}\}$ induces a partitioning



A workflow DAG

$$\mathcal{V} = \bigcup_{\mathbf{w}_i \in \mathcal{W}} \mathcal{V}_{\mathbf{w}_i} \text{ such that } i \neq j \implies \mathcal{V}_{\mathbf{w}_i} \cap \mathcal{V}_{\mathbf{w}_j} = \emptyset$$

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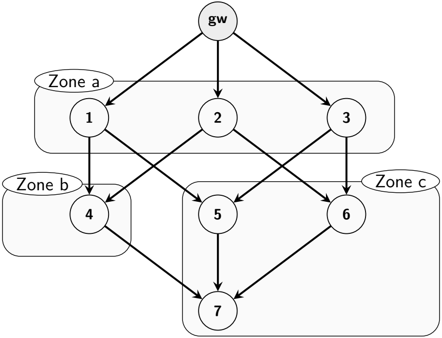

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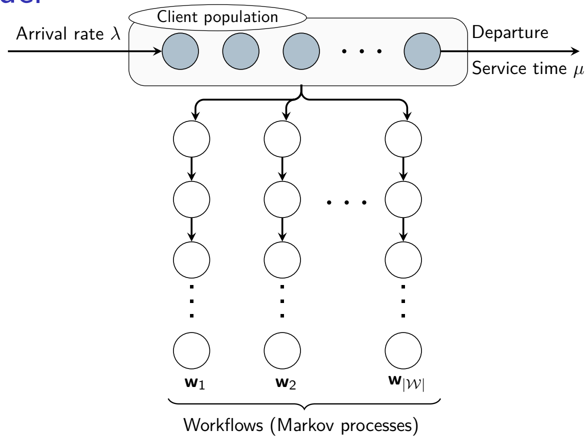
Client Model



- Homogeneous client population
- Clients arrive according to $Po(\lambda)$, Service times $Exp(\frac{1}{\mu})$
- Workflow selection: uniform
- Workflow interaction: Markov process

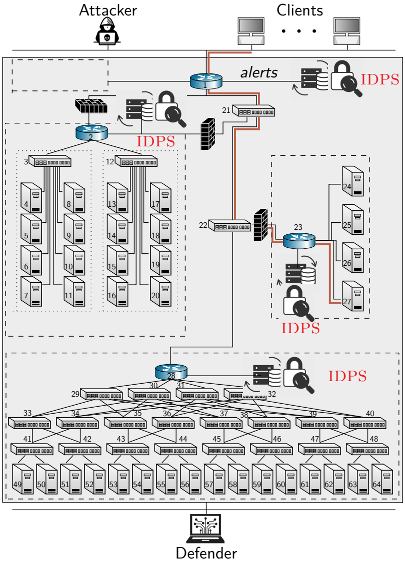
Observation Model

- IDPSs inspect network traffic and generate alert vectors:

$$\mathbf{o}_t \triangleq \left(\mathbf{o}_{t,1}, \ldots, \mathbf{o}_{t,|\mathcal{V}|}\right) \in \mathbb{N}_0^{|\mathcal{V}|}$$

$\mathbf{o}_{t,i}$ is the number of alerts related to node $i \in \mathcal{V}$ at time-step t.

- $\mathbf{o}_t = \left(\mathbf{o}_{t,1}, \ldots, \mathbf{o}_{t,|\mathcal{V}|}\right)$ is a realization of the random vector $\mathbf{O}_t$ with joint distribution Z



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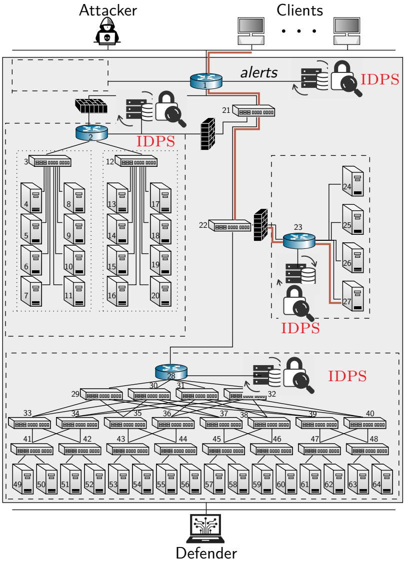
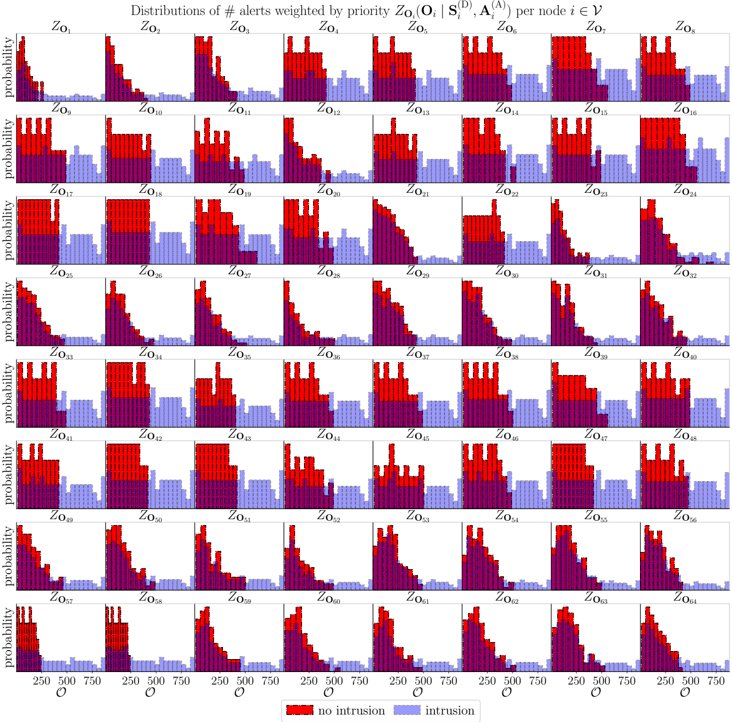
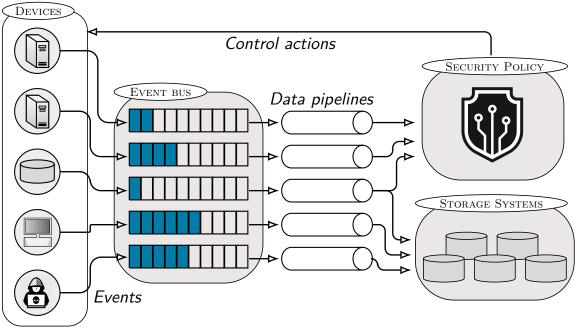



Distributions of # alerts weighted by priority $Z_{\mathbf{O}_i}(\mathbf{O}_i \mid \mathbf{S}_i^{(D)}, \mathbf{A}_i^{(A)})$ per node $i \in \mathcal{V}$
probability
$\mathcal{O}$
250 500 750
no intrusion
intrusion

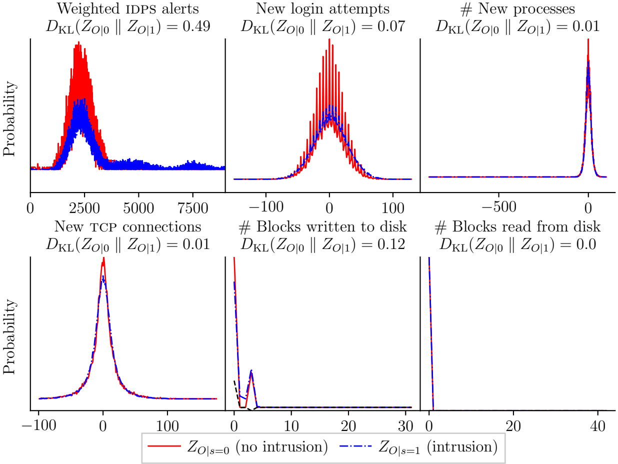
Monitoring and Telemetry



- Emulated devices run monitoring agents that **periodically push metrics to a Kafka event bus**.
- The data in the event bus is consumed by data pipelines that process the data and write to storage systems.
- The processed data is used by an automated security policy to decide on control actions to execute in the digital twin.

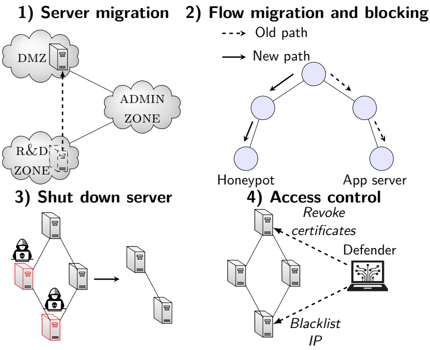
Feature Selection

- Our framework collects 100s of metrics every time-step.
- We focus on the **IDPS alert metric** as it provides the most information for detecting the type of attacks we consider.



Defender Model

- Defender action: $\mathbf{a}_t^{(D)} \in \{0, 1, 2, 3, 4\}^{|\mathcal{V}|}$
- 0 means do nothing. $1-4$ correspond to defensive actions (see fig)



- A **defender strategy** is a function $\pi_{\mathrm{D}} \in \Pi_{\mathrm{D}} : \mathcal{H}_{\mathrm{D}} \to \Delta(\mathcal{A}_{\mathrm{D}})$, where

$$\mathbf{h}_t^{(\mathrm{D})} = (\mathbf{s}_1^{(\mathrm{D})}, \mathbf{a}_1^{(\mathrm{D})}, \mathbf{o}_1, \ldots, \mathbf{a}_{t-1}^{(\mathrm{D})}, \mathbf{s}_t^{(\mathrm{D})}, \mathbf{o}_t) \in \mathcal{H}_{\mathrm{D}}$$

- Objective: (i) maintain workflows; and (ii) stop a possible intrusion:

$$J \triangleq \sum_{t=1}^{T} \gamma^{t-1} \Bigg(\underbrace{\eta \sum_{i=1}^{|\mathcal{W}|} u_{\mathrm{W}}(\mathbf{w}_i, \mathbf{s}_t)}_{\text{workflows utility}} - \underbrace{(1-\eta) \sum_{j=1}^{|\mathcal{V}|} c_{\mathrm{I}}(\mathbf{s}_{t,j}, \mathbf{a}_{t,j})}_{\text{intrusion and defense costs}} \Bigg)$$

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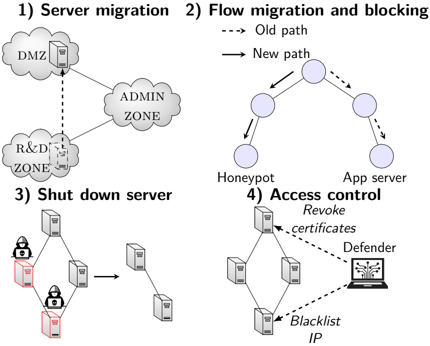


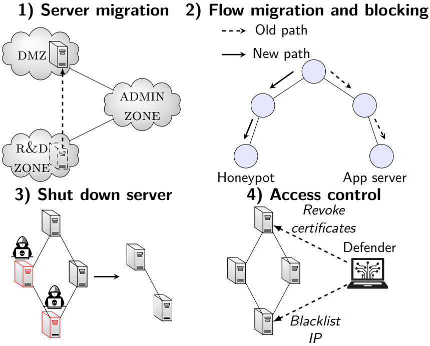
- A **defender strategy** is a function $\pi_{\mathrm{D}} \in \Pi_{\mathrm{D}} : \mathcal{H}_{\mathrm{D}} \to \Delta(\mathcal{A}_{\mathrm{D}})$, where

$$\mathbf{h}_t^{(\mathrm{D})} = (\mathbf{s}_1^{(\mathrm{D})}, \mathbf{a}_1^{(\mathrm{D})}, \mathbf{o}_1, \ldots, \mathbf{a}_{t-1}^{(\mathrm{D})}, \mathbf{s}_t^{(\mathrm{D})}, \mathbf{o}_t) \in \mathcal{H}_{\mathrm{D}}$$

- Objective: (i) maintain workflows; and (ii) stop a possible intrusion:

$$J \triangleq \sum_{t=1}^{T} \gamma^{t-1} \left(\underbrace{\eta \sum_{i=1}^{|\mathcal{W}|} u_{\mathrm{W}}(\mathbf{w}_i, \mathbf{s}_t)}_{\text{workflows utility}} - \underbrace{(1-\eta) \sum_{j=1}^{|\mathcal{V}|} c_{\mathrm{I}}(\mathbf{s}_{t,j}, \mathbf{a}_{t,j})}_{\text{intrusion and defense costs}} \right)$$

Defender Model



- Defender action: $\mathbf{a}_t^{(D)} \in \{0,1,2,3,4\}^{|\mathcal{V}|}$
- 0 means do nothing. $1-4$ correspond to defensive actions (see fig)
- A **defender strategy** is a function $\pi_{\mathrm{D}} \in \Pi_{\mathrm{D}} : \mathcal{H}_{\mathrm{D}} \to \Delta(\mathcal{A}_{\mathrm{D}})$, where

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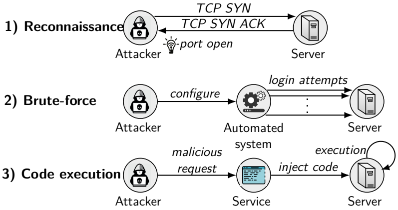
Attacker Model

- Attacker action: $\mathbf{a}_t^{(A)} \in \{0, 1, 2, 3\}^{|\mathcal{V}|}$
- 0 means do nothing. $1 - 3$ correspond to attacks (see fig)



- An **attacker strategy** is a function $\pi_A \in \Pi_A : \mathcal{H}_A \rightarrow \Delta(\mathcal{A}_A)$, where $\mathcal{H}_A$ is the space of all possible attacker histories

$$\mathbf{h}_t^{(A)} = (\mathbf{s}_1^{(A)}, \mathbf{a}_1^{(A)}, \mathbf{o}_1, \ldots, \mathbf{a}_{t-1}^{(A)}, \mathbf{s}_t^{(A)}, \mathbf{o}_t) \in \mathcal{H}_A$$

- Objective: (i) disrupt workflows; and (ii) **compromise nodes**:

$$-J$$

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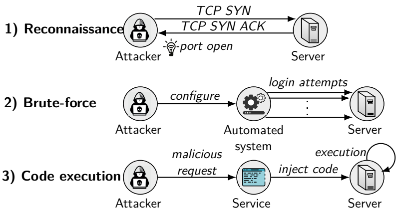

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$$\mathbf{h}_t^{(\mathrm{A})} = (\mathbf{s}_1^{(\mathrm{A})}, \mathbf{a}_1^{(\mathrm{A})}, \mathbf{o}_1, \ldots, \mathbf{a}_{t-1}^{(\mathrm{A})}, \mathbf{s}_t^{(\mathrm{A})}, \mathbf{o}_t) \in \mathcal{H}_{\mathrm{A}}$$

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Attacker Model

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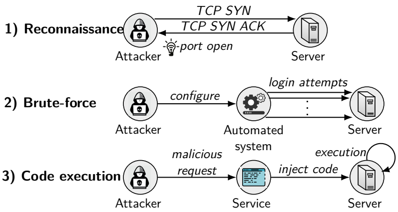

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- Objective: (i) disrupt workflows; and (ii) **compromise nodes**:

$$-J$$

The Intrusion Response Problem

$$
\begin{aligned}
\underset{\pi_{\mathrm{D}} \in \Pi_{\mathrm{D}}}{\text{maximize}} \ \underset{\pi_{\mathrm{A}} \in \Pi_{\mathrm{A}}}{\text{minimize}} \ & \mathbb{E}_{(\pi_{\mathrm{D}}, \pi_{\mathrm{A}})}\left[J\right] && && \text{(1a)} \\
\text{subject to } & \mathbf{s}^{(\mathrm{D})}_{t+1} \sim f_{\mathrm{D}}(\cdot \mid \mathbf{A}^{(\mathrm{D})}_t, \mathbf{A}^{(\mathrm{D})}_t) && \forall t && \text{(1b)} \\
& \mathbf{s}^{(\mathrm{A})}_{t+1} \sim f_{\mathrm{A}}(\cdot \mid \mathbf{S}^{(\mathrm{A})}_t, \mathbf{A}_t) && \forall t && \text{(1c)} \\
& \mathbf{o}_{t+1} \sim Z(\cdot \mid \mathbf{S}^{(\mathrm{D})}_{t+1}, \mathbf{A}^{(\mathrm{A})}_t) && \forall t && \text{(1d)} \\
& \mathbf{a}^{(\mathrm{A})}_t \sim \pi_{\mathrm{A}}(\cdot \mid \mathbf{H}^{(\mathrm{A})}_t), \ \mathbf{a}^{(\mathrm{A})}_t \in \mathcal{A}_{\mathrm{A}}(\mathbf{s}_t) && \forall t && \text{(1e)} \\
& \mathbf{a}^{(\mathrm{D})}_t \sim \pi_{\mathrm{D}}(\cdot \mid \mathbf{H}^{(\mathrm{D})}_t), \ \mathbf{a}^{(\mathrm{D})}_t \in \mathcal{A}_{\mathrm{D}} && \forall t && \text{(1f)}
\end{aligned}
$$

where $\mathbb{E}_{(\pi_{\mathrm{D}}, \pi_{\mathrm{A}})}$ denotes the expectation of the random vectors $(\mathbf{S}_t, \mathbf{O}_t, \mathbf{A}_t)_{t \in \{1,\ldots,T\}}$ under the strategy profile $(\pi_{\mathrm{D}}, \pi_{\mathrm{A}})$.

(1) can be formulated as a zero-sum Partially Observed Stochastic Game with Public Observations (a PO-POSG):

$$
\Gamma = \langle \mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (\mathcal{A}_i)_{i \in \mathcal{N}}, (f_i)_{i \in \mathcal{N}}, u, \gamma, (\mathbf{b}^{(i)}_1)_{i \in \mathcal{N}}, \mathcal{O}, Z \rangle
$$

Existence of a Solution

Theorem

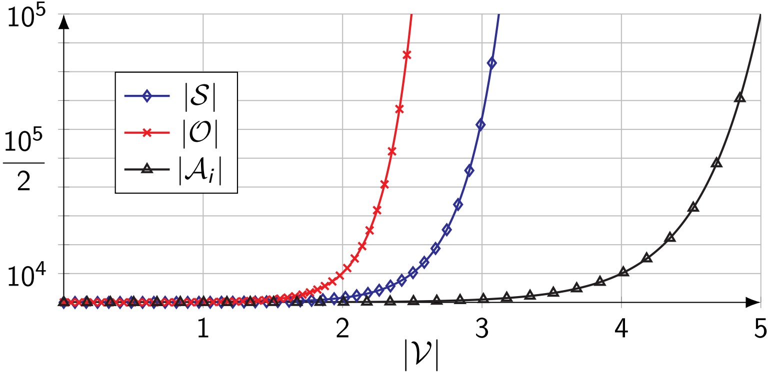
Given the PO-POSG Γ *(2), the following holds:*

(A) Γ *has a mixed Nash equilibrium and a value function* $V^* : \mathcal{B}_{\mathrm{D}} \times \mathcal{B}_{\mathrm{A}} \rightarrow \mathbb{R}$ *that maps each possible initial pair of belief states* $(\mathbf{b}_1^{(\mathrm{D})}, \mathbf{b}_1^{\mathrm{A}})$ *to the expected utility of the defender in the equilibrium.*

(B) *For each strategy pair* $(\pi_{\mathrm{A}}, \pi_{\mathrm{D}}) \in \Pi_{\mathrm{A}} \times \Pi_{\mathrm{D}}$*, the best response sets* $B_{\mathrm{D}}(\pi_{\mathrm{A}})$ *and* $B_{\mathrm{A}}(\pi_{\mathrm{D}})$ *are non-empty and correspond to optimal strategies in two Partially Observed Markov Decision Processes (*POMDP*s):* $\mathscr{M}^{(\mathrm{D})}$ *and* $\mathscr{M}^{(\mathrm{A})}$*. Further, a pair of pure best response strategies* $(\tilde{\pi}_{\mathrm{D}}, \tilde{\pi}_{\mathrm{A}}) \in B_{\mathrm{D}}(\pi_{\mathrm{A}}) \times B_{\mathrm{A}}(\pi_{\mathrm{D}})$ *and a pair of value functions* $(V^*_{\mathrm{D},\pi_{\mathrm{A}}}, V^*_{\mathrm{A},\pi_{\mathrm{D}}})$ *exist.*

The Curse of Dimensionality

- While (1) has a solution (i.e the game Γ has a value (Thm 1)), computing it is intractable since the state, action, and observation spaces of the game **grow exponentially** with $|\mathcal{V}|$.



Growth of $|\mathcal{S}|$, $|\mathcal{O}|$, and $|\mathcal{A}_i|$ in function of the number of nodes $|\mathcal{V}|$

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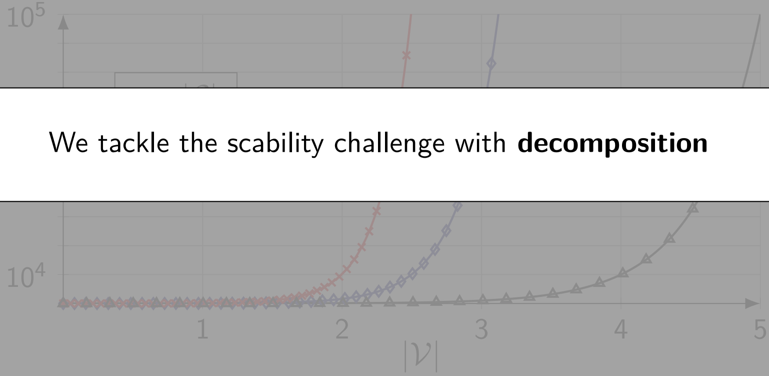
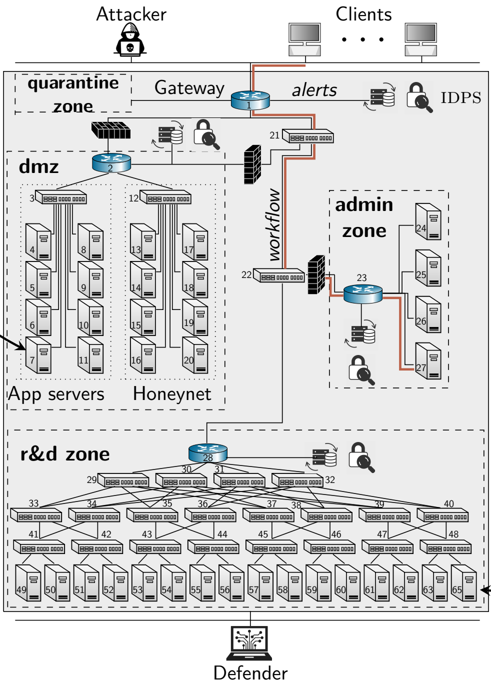


We tackle the scability challenge with **decomposition**

Growth of $|\mathcal{S}|$, $|\mathcal{O}|$, and $|\mathcal{A}_i|$ in function of the number of nodes $|\mathcal{V}|$

Intuitively..



The optimal action here...

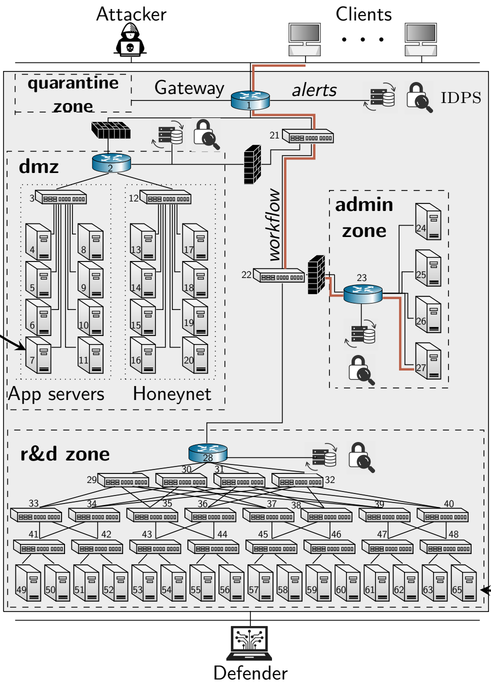
Does not directly depend on the state or action of a node down here

Intuitively..

The optimal action here...

But they are not completely independent either.

How can we exploit this structure?



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System Decomposition

To avoid explicitly enumerating the very large state, observation, and action spaces of Γ, we exploit three structural properties.

1. **Additive structure across workflows.**
   - The game decomposes into additive subgames on the workflow-level, which means that the strategy for each subgame can be optimized independently
2. **Optimal substructure within a workflow.**
   - The subgame for each workflow decomposes into subgames on the node-level that satisfy the *optimal substructure* property
3. **Threshold properties of local defender strategies.**
   - The optimal node-level strategies for the defender exhibit threshold structures, which means that they can be estimated efficiently

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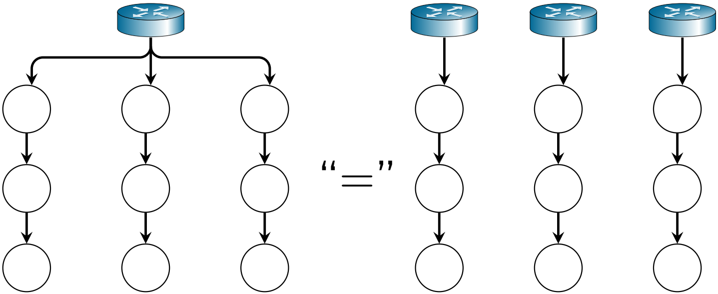
Additive Structure Across Workflows (Intuition)



- If there is no path between i and j in $\mathcal{G}$, then i and j are **independent** in the following sense:
  - Compromising i has no affect on the state of j.
  - Compromising i does not make it harder or easier to compromise j.
  - Compromising i does not affect the service provided by j.
  - Defending i does not affect the state of j.
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Additive Structure Across Workflows

Definition (Transition independence)

A set of nodes $\mathcal{Q}$ are transition independent iff the transition probabilities factorize as

$$f(\mathbf{S}_{t+1} \mid \mathbf{S}_t, \mathbf{A}_t) = \prod_{i \in \mathcal{Q}} f(\mathbf{S}_{t+1,i} \mid \mathbf{S}_{t,i}, \mathbf{A}_{t,i})$$

Definition (Utility independence)

A set of nodes $\mathcal{Q}$ are utility independent iff there exists functions $u_1, \ldots, u_{|\mathcal{Q}|}$ such that the utility function u decomposes as

$$u(\mathbf{S}_t, \mathbf{A}_t) = f(u_1(\mathbf{S}_{t,1}, \mathbf{A}_{t,1}), \ldots, u_1(\mathbf{S}_{t,|\mathcal{Q}|}, \mathbf{A}_{t,\mathcal{Q}}))$$

and

$$u_i \leq u_i' \iff f(u_1, \ldots, u_i, \ldots, u_{|\mathcal{Q}|}) \leq f(u_1, \ldots, u_i', \ldots, u_{|\mathcal{Q}|})$$

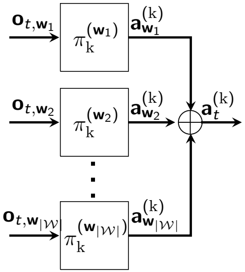
Additive Structure Across Workflows

Theorem (Additive structure across workflows)

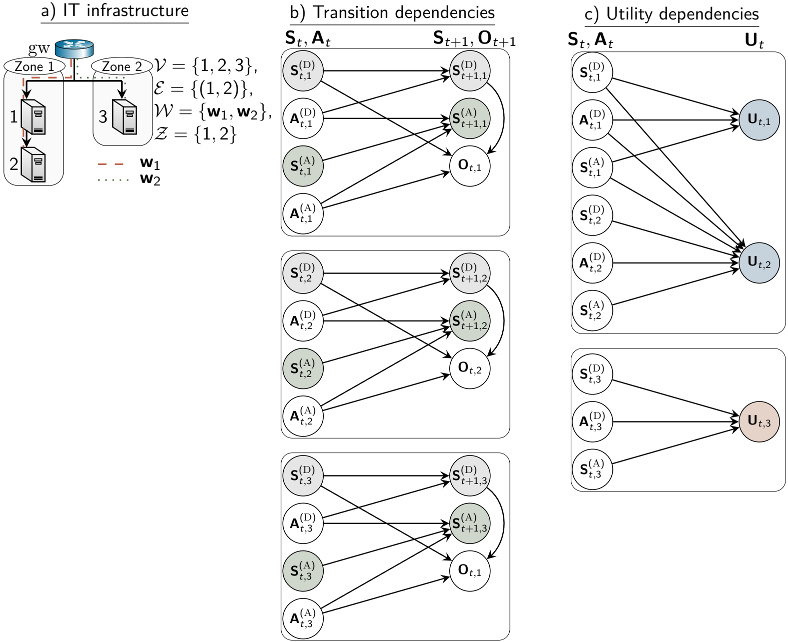
(A) All nodes $\mathcal{V}$ in the game Γ are transition independent.
(B) If there is no path between i and j in the topology graph $\mathcal{G}$, then i and j are utility independent.

Corollary

Γ decomposes into $|\mathcal{W}|$ additive subproblems that can be solved independently and in parallel.



Additive Structure Across Workflows: Minimal Example



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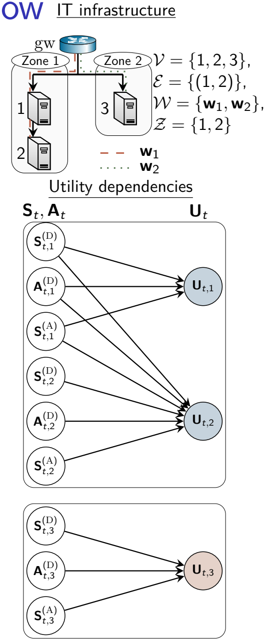
Optimal Substructure Within a Workflow

- Nodes in the same workflow are utility dependent.
- $\implies$ Locally-optimal strategies for each node **can not** simply be added together to obtain an optimal strategy for the workflow.
- However, the locally-optimal strategies satisfy the optimal substructure property.
- $\implies$ there exists an algorithm for constructing an optimal workflow strategy from locally-optimal strategies for each node.



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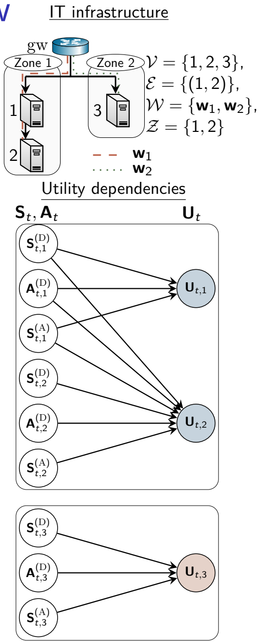


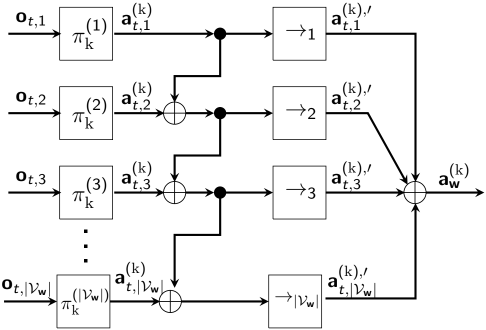
Algorithm for Combining Locally-Optimal Node Strategies into Optimal Workflow Strategies

Algorithm 1: Algorithm for combining local strategies

Input: Γ*:* the game,
$\boldsymbol{\pi}_k$: a vector with local strategies

Output: (π_D, π_A)*:* global game strategies

Algorithm $\textsc{composite-strategy}(\Gamma, \boldsymbol{\pi}_k)$
  for *player* $k \in \mathcal{N}$ **do**
    $\pi_k \leftarrow \lambda\ (\mathbf{s}_t^{(k)}, \mathbf{b}_t^{(k)})$
      $\mathbf{a}_t^{(k)} = ()$
      for *workflow* $\mathbf{w} \in \mathcal{W}$ **do**
        for *node* $i \in \textsc{topological-sort}(\mathcal{V}_{\mathbf{w}})$ **do**
          $\mathbf{a}_t^{(k,i)} \leftarrow \boldsymbol{\pi}_k^{(i)}(\mathbf{s}_t^{(k)}, \mathbf{b}_t^{(k)})$
          if $\mathrm{gw} \not\rightarrow_t^{\mathbf{a}_t^{(k)}} i$ **then**
            $\mathbf{a}_t^{(k,i)} \leftarrow \bot$
          end
          $\mathbf{a}_t^{(k)} = \mathbf{a}_t^{(k)} \oplus \mathbf{a}_t^{(k,i)}$
        end
      end
      return $\mathbf{a}_t^{(k)}$
  end
  return (π_D, π_A)



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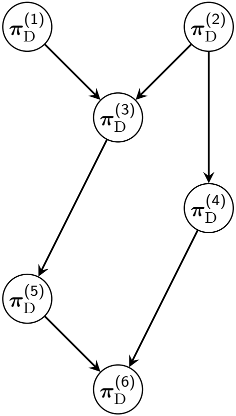
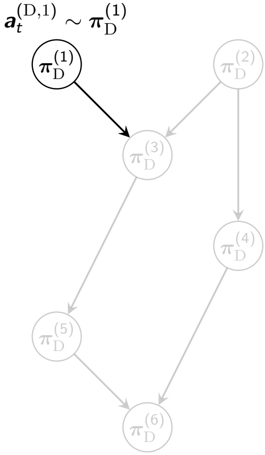


$(\boldsymbol{\pi}_D^{(i)})_{i\in\mathcal{V}_{\mathbf{w}}}$: local strategies in the same workflow $\mathbf{w} \in \mathcal{W}$

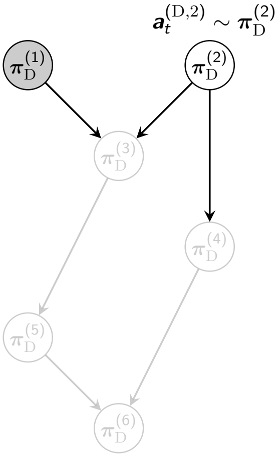
Algorithm for Combining Locally-Optimal Node Strategies into Optimal Workflow Strategies



Workflow action:
$(\boldsymbol{a}_t^{(\mathrm{D},1)})$

Step 1; select action for node 1 according to its local strategy

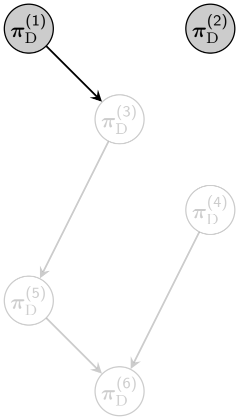
Algorithm for Combining Locally-Optimal Node Strategies into Optimal Workflow Strategies



Workflow action:
$(\boldsymbol{a}_t^{(\mathrm{D},1)}, \boldsymbol{a}_t^{(\mathrm{D},2)})$

Step 2; update the topology based on the previous local action;
select action $a = 0$ for unreachable nodes;
move to the next node in the topological ordering (i.e. 2);
select the action for the next node according to its local strategy.

Algorithm for Combining Locally-Optimal Node Strategies into Optimal Workflow Strategies



Workflow action:
$(\boldsymbol{a}_t^{(\mathrm{D},1)}, \boldsymbol{a}_t^{(\mathrm{D},2)})$

Step 3; update the topology based on the previous local action;
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move to the next node in the topological ordering (i.e. 3);
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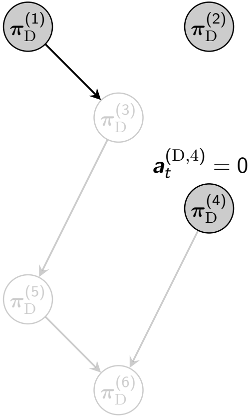

Workflow action:
$(\boldsymbol{a}_t^{(\mathrm{D},1)}, \boldsymbol{a}_t^{(\mathrm{D},2)}, \cdot, 0)$

Step 3; update the topology based on the previous local action;
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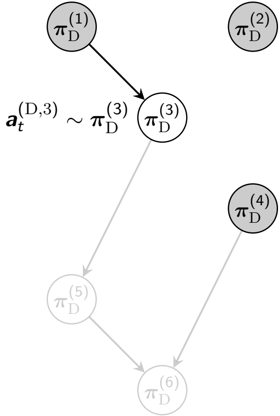

Workflow action:
$(a_t^{(D,1)}, a_t^{(D,2)}, a_t^{(D,3)}, 0)$

Step 3; update the topology based on the previous local action;
select action $a = 0$ for unreachable nodes;
move to the next node in the topological ordering (i.e. 3);
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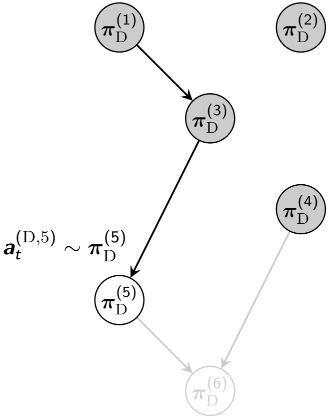
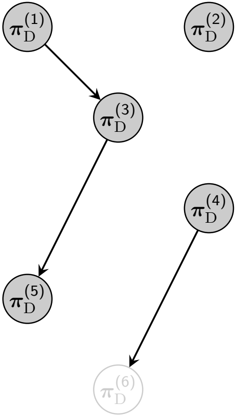


Workflow action:
$(\boldsymbol{a}_t^{(\mathrm{D},1)}, \boldsymbol{a}_t^{(\mathrm{D},2)}, \boldsymbol{a}_t^{(\mathrm{D},3)}, 0, \boldsymbol{a}_t^{(\mathrm{D},5)})$

Step 4; update the topology based on the previous local action;
select action $a = 0$ for unreachable nodes;
move to the next node in the topological ordering (i.e. 5);
select the action for the next node according to its local strategy.

Algorithm for Combining Locally-Optimal Node Strategies into Optimal Workflow Strategies



Workflow action:
$(\boldsymbol{a}_t^{(\mathrm{D},1)}, \boldsymbol{a}_t^{(\mathrm{D},2)}, \boldsymbol{a}_t^{(\mathrm{D},3)}, 0, \boldsymbol{a}_t^{(\mathrm{D},5)})$

Step 5; update the topology based on the previous local action;
select action $a = 0$ for unreachable nodes;

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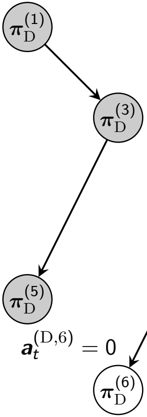
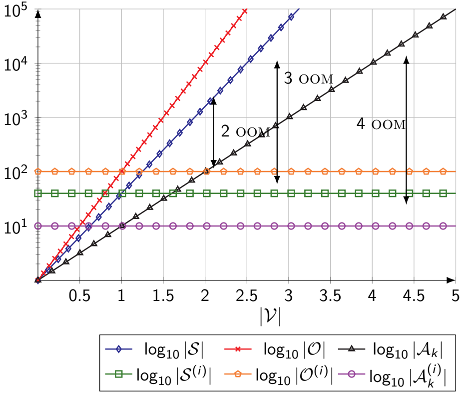


Workflow action:
$(\boldsymbol{a}_t^{(\mathrm{D},1)}, \boldsymbol{a}_t^{(\mathrm{D},2)}, \boldsymbol{a}_t^{(\mathrm{D},3)}, 0, \boldsymbol{a}_t^{(\mathrm{D},5)}, 0)$

Step 5; update the topology based on the previous local action;
select action $a = 0$ for unreachable nodes;

Computational Benefits of Decomposition

- $\therefore$ we can obtain an optimal (best response) strategy for the full game Γ by combining the solutions to $\mathcal{V}$ simpler subproblems that can be solved **in parallel** and have significantly smaller state, observation, and action spaces.



Space complexity comparison between the full game and the decomposed game.

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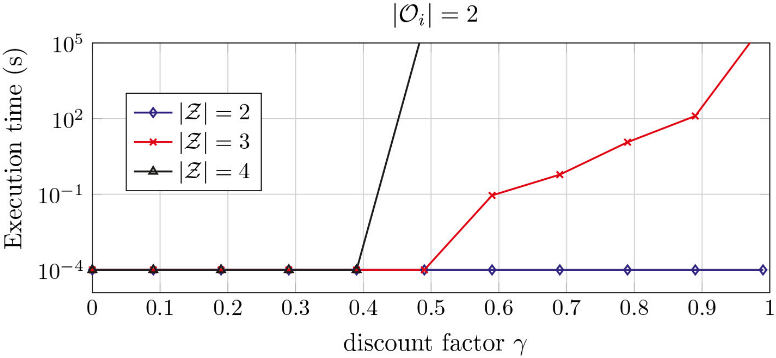
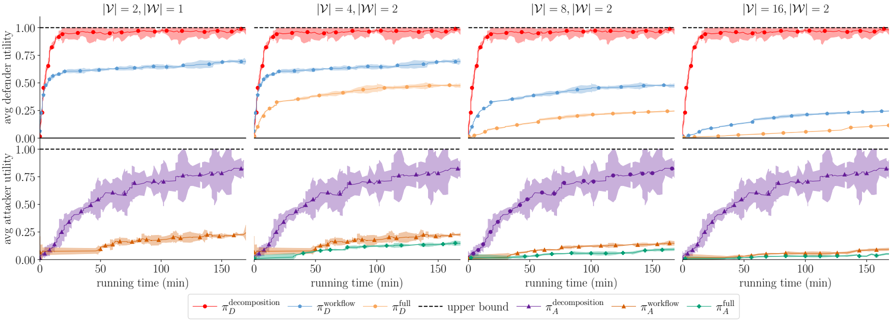
Can we Solve the Local Problems with Dynamic Programming?

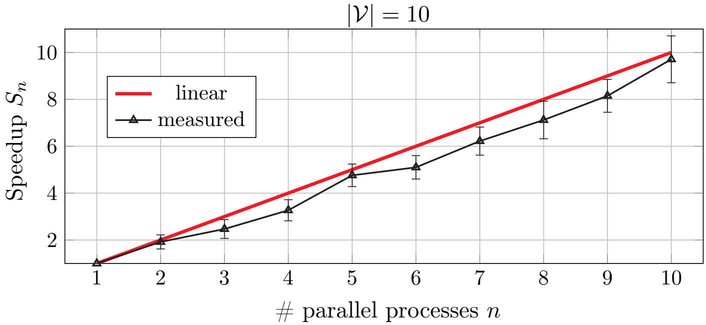


Scalable learning through decomposition (Simulation)



Learning curves obtained during training of PPO to find best response strategies against randomized opponents; red, purple, blue and brown curves relate to decomposed strategies; the orange and green curves relate to the non-decomposed strategies.

Scalable learning through decomposition (Simulation)



Speedup of completion time when computing best response strategies for the decomposed game with $|\mathcal{V}| = 10$ nodes and different number of parallel processes; the subproblems in the decomposition are split evenly across the processes; let T_n denote the completion time when using n processes, the speedup is then calculated as $S_n = \frac{T_1}{T_n}$; the error bars indicate standard deviations from 3 measurements.

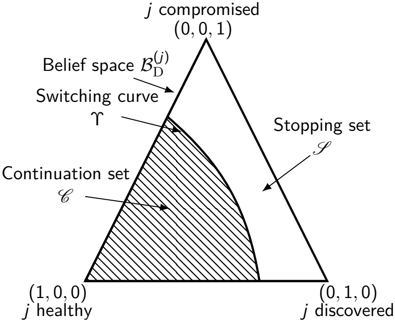
Threshold Properties of Local Defender Strategies.

- The local problem of the defender can be decomposed in the temporal domain as

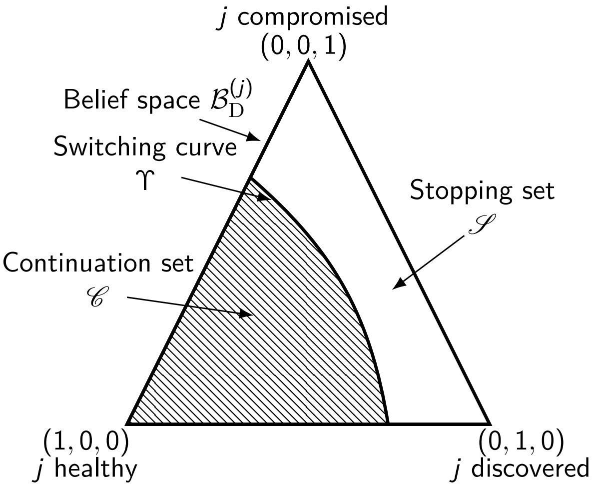
$$\max_{\pi_{\mathrm{D}}} \sum_{t=1}^{T} J = \max_{\pi_{\mathrm{D}}} \sum_{t=1}^{\tau_1} J_1 + \sum_{t=1}^{\tau_2} J_2 + \dots \tag{2}$$

where $\tau_1, \tau_2, \dots$ are stopping times.

- $\implies$ (1) selection of defensive actions is simplified; and (2) the optimal stopping times are given by a threshold strategy that can be estimated efficiently:



Threshold Properties of Local Defender Strategies.



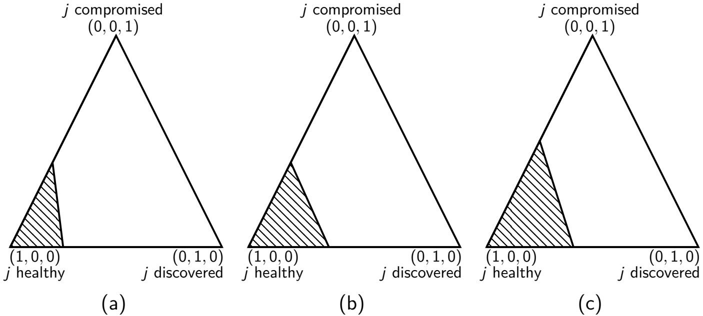
- A node can be in three attack states $s_t^{(\mathrm{A})}$: Healthy, Discovered, Compromised.
- The defender has a belief state $\mathbf{b}_t^{(\mathrm{D})}$

Threshold Properties of Local Defender Strategies.

We estimate the optimal switching curves using a linear approximation

$$\pi_{\mathrm{D}}(\mathbf{b}^{(\mathrm{D})}) = \begin{cases} \text{Stop} & \text{if } \begin{bmatrix} 0 & 1 & \boldsymbol{\theta}^T \end{bmatrix} \begin{bmatrix} \mathbf{b}^{(\mathrm{D})} \\ -1 \end{bmatrix} < 0 \\ \text{Continue} & \text{otherwise} \end{cases} \tag{3}$$

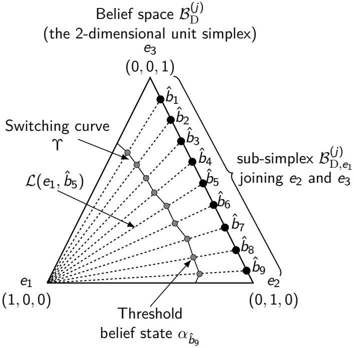
$$\text{subject to } \boldsymbol{\theta} \in \mathbb{R}^2,\ \boldsymbol{\theta}_2 > 0 \text{ and } \boldsymbol{\theta}_1 \geq 1 \tag{4}$$



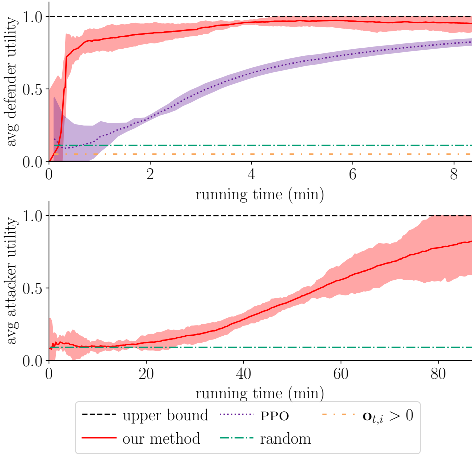
Examples of learned linear switching curves.

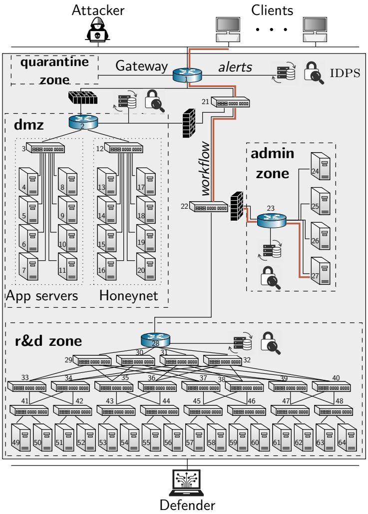
Proof Sketch (Threshold Properties)

- Let $\mathcal{L}(e_1, \hat{b})$ denote the line segment that starts at the belief state $e_1 = (1, 0, 0)$ and ends at $\hat{b}$, where $\hat{b}$ is in the sub-simplex that joins e_2 and e_3.
- All beliefs on $\mathcal{L}(e_1, \hat{b})$ are totally ordered according to the Monotone Likelihood Ratio (MLR) order. $\implies$ a threshold belief state $\alpha_{\hat{b}} \in \mathcal{L}(e_1, \hat{b})$ exists where the optimal strategy switches from C to S.
- Since the entire belief space can be covered by the union of lines $\mathcal{L}(e_1, \hat{b})$, the threshold belief states $\alpha_{\hat{b}_1}, \alpha_{\hat{b}_2}, \ldots$ yield a switching curve Υ.

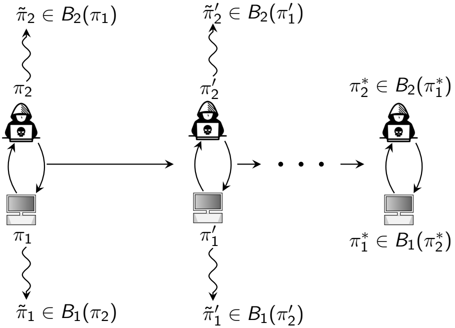


Learning Best Responses for the Target Infrastructure (Simulation)





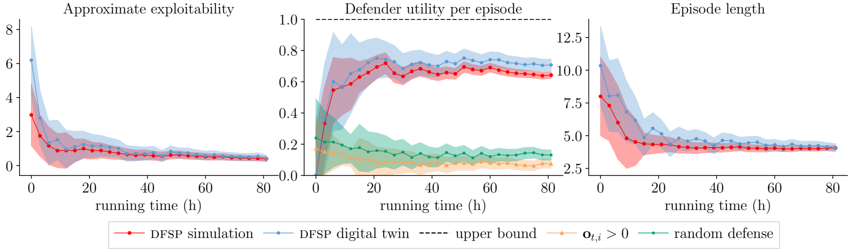
Decompositional Fictitious Play (DFSP) to Approximate an Equilibrium



Fictitious play: iterative averaging of best responses.

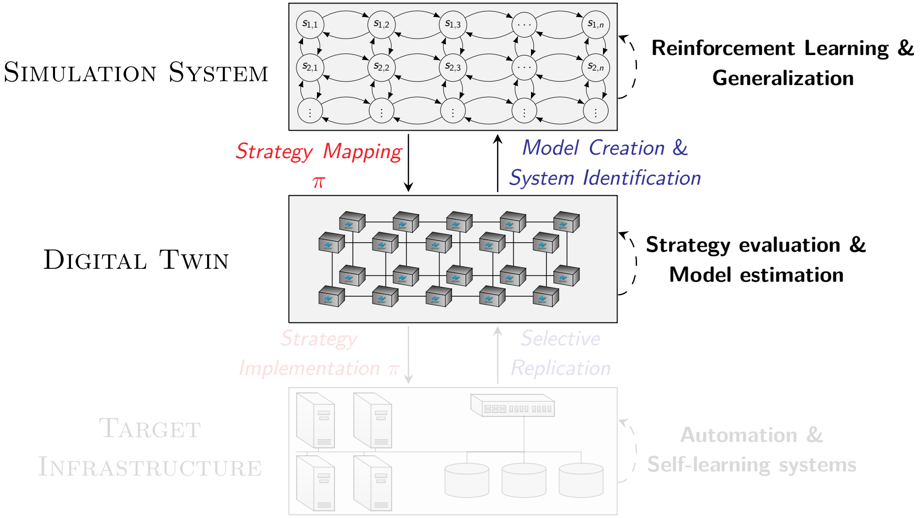
- Learn best response strategies iteratively through the parallel solving of subgames in the decomposition
- Average best responses to approximate the equilibrium

Learning Equilibrium Strategies

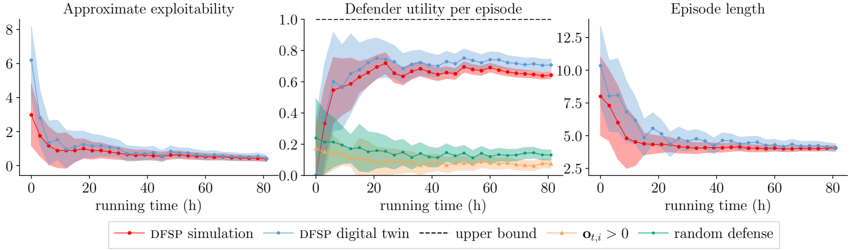


Learning curves obtained during training of DFSP to find optimal (equilibrium) strategies in the intrusion response game; red and blue curves relate to DFSP; black, orange and green curves relate to baselines.

Evaluation in the Digital Twin

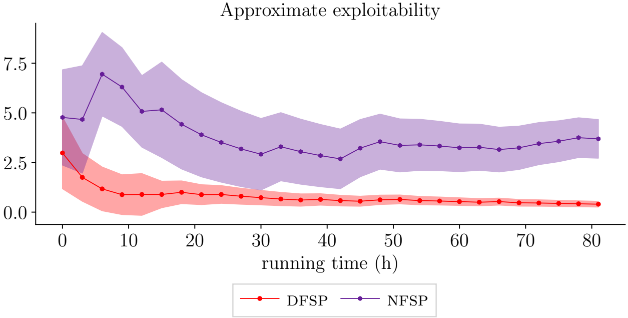


Learning Equilibrium Strategies



Learning curves obtained during training of DFSP to find optimal (equilibrium) strategies in the intrusion response game; red and blue curves relate to DFSP; black, orange and green curves relate to baselines.

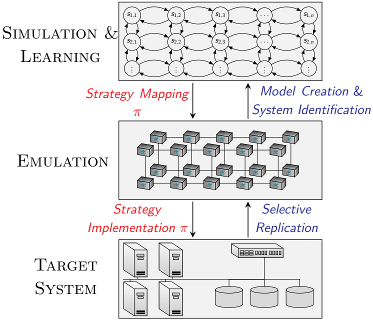
Learning Equilibrium Strategies (Comparison against NFSP)



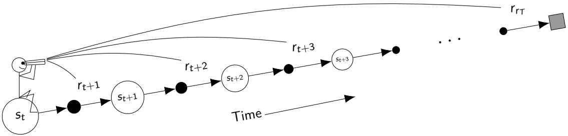
Learning curves obtained during training of DFSP and NFSP to find optimal (equilibrium) strategies in the intrusion response game; the red curve relate to DFSP and the purple curve relate to NFSP; all curves show simulation results.

Conclusions

- We develop a *framework* to automatically learn security strategies.
- We apply the method to an **intrusion response use case**.
- We design a novel decompositional approach to find near-optimal intrusion responses for large-scale IT infrastructures.
- We show that the decomposition reduces both the computational complexity of finding effective strategies, and the sample complexity of learning a system model by several orders of magnitude.



Current and Future Work



1. **Extend use case**
   - Heterogeneous client population
   - Extensive threat model of the attacker
2. **Extend solution framework**
   - Model-predictive control
   - Rollout-based techniques
   - Extend system identification algorithm
3. **Extend theoretical results**
   - Exploit symmetries and causal structure