Scalable Learning of Intrusion Response through Recursive Decomposition

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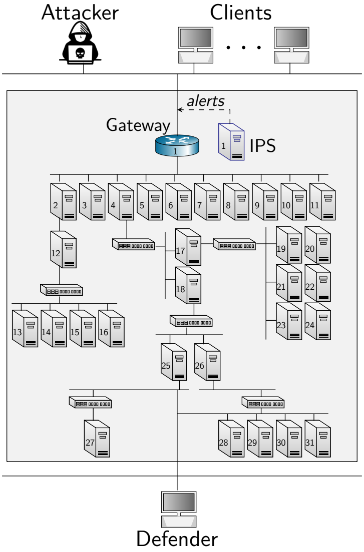
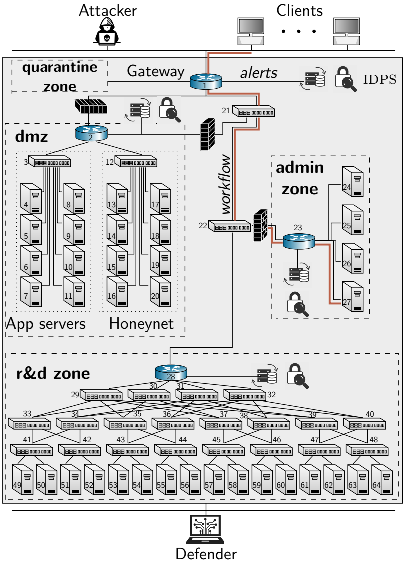
Use Case: Intrusion Response

- A **defender** owns an infrastructure
  - Consists of connected components
  - Components run network services
  - Defender defends the infrastructure by monitoring and active defense
  - Has partial observability
- An **attacker** seeks to intrude on the infrastructure
  - Has a partial view of the infrastructure
  - Wants to compromise specific components
  - Attacks by reconnaissance, exploitation and pivoting



System Model

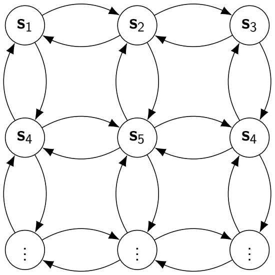
- $\mathcal{G} = \langle \{\text{gw}\} \cup \mathcal{V}, \mathcal{E} \rangle$: directed tree representing the virtual infrastructure
- $\mathcal{V}$: finite set of virtual components.
- $\mathcal{E}$: finite set of component dependencies.
- $\mathcal{Z}$: finite set of zones.



State Space

- Each $i \in \mathcal{V}$ has a state

$$\boldsymbol{v}_{t,i} = (\underbrace{v_{t,i}^{(Z)}}_{\text{D}}, \underbrace{v_{t,i}^{(I)}, v_{t,i}^{(R)}}_{\text{A}})$$



- System state $\mathbf{s}_t = (\mathbf{v}_{t,i})_{i \in \mathcal{V}} \sim \mathbf{S}_t$.
- Markovian time-homogeneous dynamics:

$$\mathbf{s}_{t+1} \sim f(\cdot \mid \mathbf{S}_t, \mathbf{A}_t)$$

$\mathbf{A}_t = (\mathbf{A}_t^{(A)}, \mathbf{A}_t^{(D)})$ are the actions.

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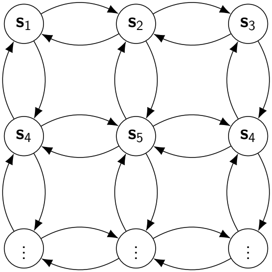

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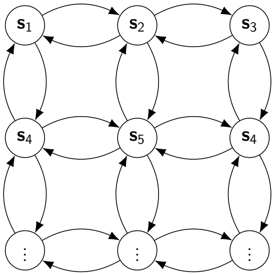

Workflows

- Services are connected into **workflows** $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{|\mathcal{W}|}\}$.

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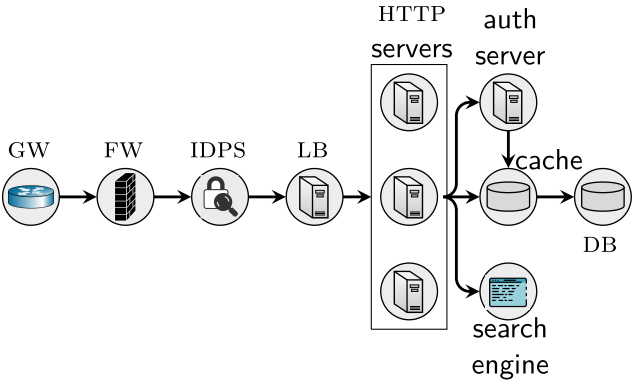
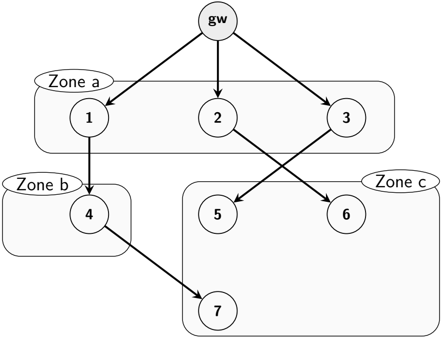


Dependency graph of an example workflow representing a web application; GW, FW, IDPS, LB, and DB are acronyms for gateway, firewall, intrusion detection and prevention system, load balancer, and database, respectively.

Workflows

- Services are connected into **workflows** $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{|\mathcal{W}|}\}$.
- Each $\mathbf{w} \in \mathcal{W}$ is realized as a subtree $\mathcal{G}_{\mathbf{w}} = \langle \{\mathrm{gw}\} \cup \mathcal{V}_{\mathbf{w}}, \mathcal{E}_{\mathbf{w}} \rangle$ of $\mathcal{G}$
- $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{|\mathcal{W}|}\}$ induces a partitioning



A workflow tree

$$\mathcal{V} = \bigcup_{\mathbf{w}_i \in \mathcal{W}} \mathcal{V}_{\mathbf{w}_i} \text{ such that } i \neq j \implies \mathcal{V}_{\mathbf{w}_i} \cap \mathcal{V}_{\mathbf{w}_j} = \emptyset$$

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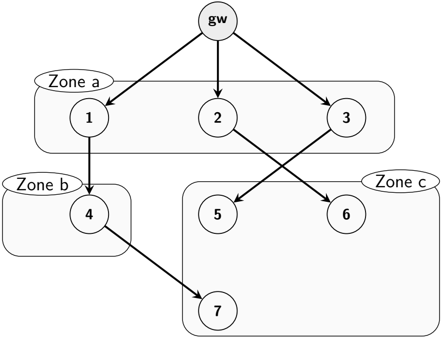

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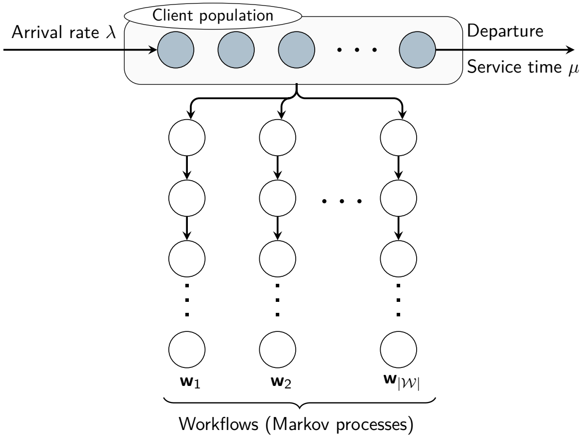
Clients



- Homogeneous client population
- Clients arrive according to $Po(\lambda)$, Service times $Exp(\frac{1}{\mu})$
- Workflow selection: uniform
- Workflow interaction: Markov process

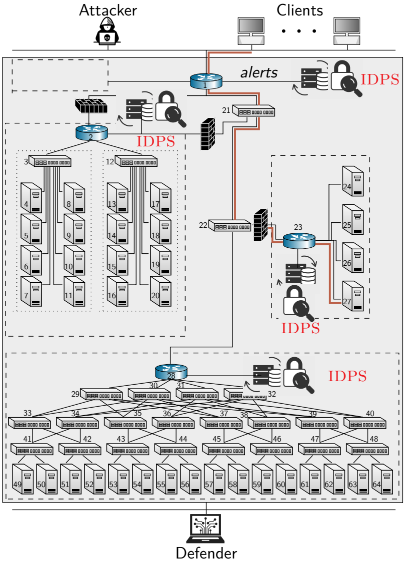
Observations

- IDPSs inspect network traffic and generate alert vectors:

$$\mathbf{o}_t \triangleq \left(\mathbf{o}_{t,1}, \ldots, \mathbf{o}_{t,|\mathcal{V}|}\right) \in \mathbb{N}_0^{|\mathcal{V}|}$$

$\mathbf{o}_{t,i}$ is the number of alerts related to node $i \in \mathcal{V}$ at time-step t.

- $\mathbf{o}_t = \left(\mathbf{o}_{t,1}, \ldots, \mathbf{o}_{t,|\mathcal{V}|}\right)$ is a realization of the random vector $\mathbf{O}_t$ with joint distribution Z



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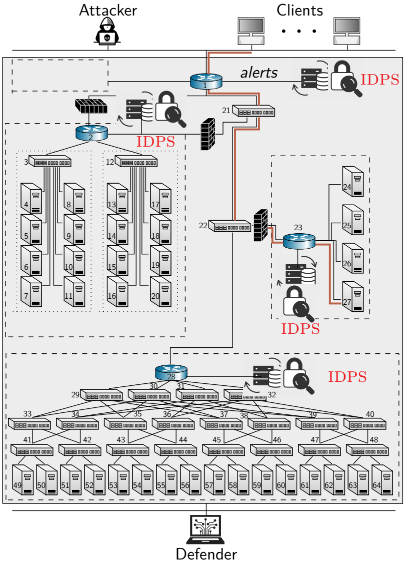
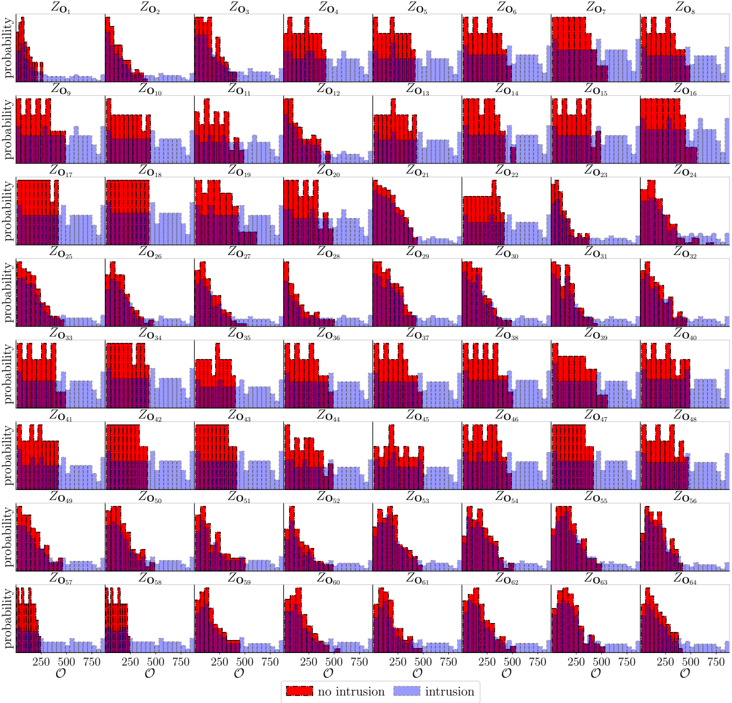
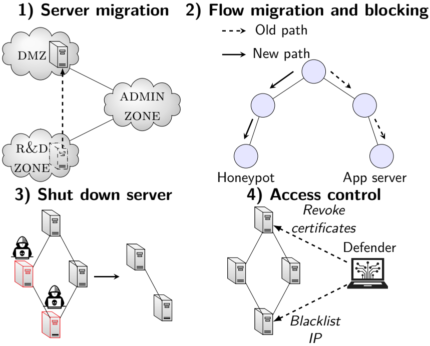


Distributions of # alerts weighted by priority $Z_{\mathbf{O}_i}(\mathbf{O}_i \mid \mathbf{S}_i^{(\mathrm{D})}, \mathbf{A}_i^{(\mathrm{A})})$ per node $i \in \mathcal{V}$



Defender

- Defender action: $\mathbf{a}_t^{(D)} \in \{0,1,2,3,4\}^{|\mathcal{V}|}$
- 0 means do nothing. $1-4$ correspond to defensive actions (see fig)



- A **defender strategy** is a function $\pi_{\mathrm{D}} \in \Pi_{\mathrm{D}} : \mathcal{H}_{\mathrm{D}} \rightarrow \Delta(\mathcal{A}_{\mathrm{D}})$, where

$$\mathbf{h}_t^{(\mathrm{D})} = (\mathbf{s}_1^{(\mathrm{D})}, \mathbf{a}_1^{(\mathrm{D})}, \mathbf{o}_1, \ldots, \mathbf{a}_{t-1}^{(\mathrm{D})}, \mathbf{s}_t^{(\mathrm{D})}, \mathbf{o}_t) \in \mathcal{H}_{\mathrm{D}}$$

- Objective: (*i*) maintain workflows; and (*ii*) stop a possible intrusion:

$$J \triangleq \sum_{t=1}^{T} \gamma^{t-1} \Bigg(\underbrace{\eta \sum_{i=1}^{|\mathcal{W}|} u_{\mathrm{W}}(\mathbf{w}_i, \mathbf{s}_t)}_{\text{workflows utility}} - \underbrace{(1-\eta) \sum_{j=1}^{|\mathcal{V}|} c_{\mathrm{I}}(\mathbf{s}_{t,j}, \mathbf{a}_{t,j})}_{\text{intrusion and defense costs}} \Bigg)$$

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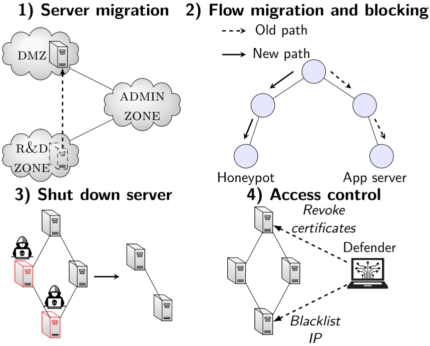

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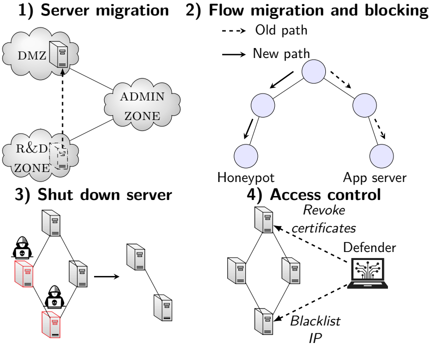

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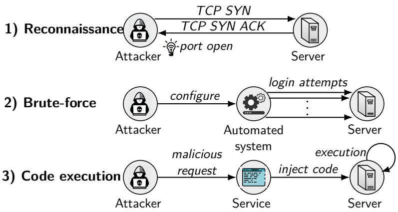
Attacker

- Attacker action: $\mathbf{a}_t^{(A)} \in \{0, 1, 2, 3\}^{|\mathcal{V}|}$
- 0 means do nothing. $1 - 3$ correspond to attacks (see fig)



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$$\mathbf{h}_t^{(A)} = (\mathbf{s}_1^{(A)}, \mathbf{a}_1^{(A)}, \mathbf{o}_1, \ldots, \mathbf{a}_{t-1}^{(A)}, \mathbf{s}_t^{(A)}, \mathbf{o}_t) \in \mathcal{H}_A$$

- Objective: (i) disrupt workflows; and (ii) **compromise nodes**:

$$-J$$

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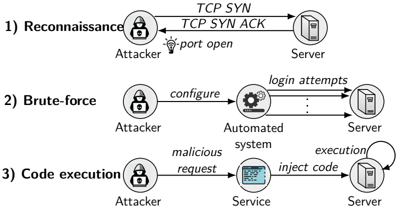

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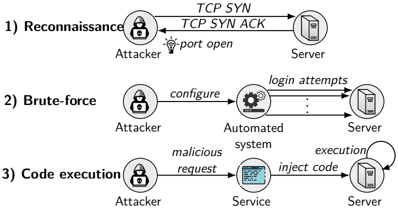

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The Intrusion Response Problem

$$\underset{\pi_{\mathrm{D}} \in \Pi_{\mathrm{D}}}{\text{maximize}} \ \underset{\pi_{\mathrm{A}} \in \Pi_{\mathrm{A}}}{\text{minimize}} \ \mathbb{E}_{(\pi_{\mathrm{D}}, \pi_{\mathrm{A}})}[J] \tag{1a}$$

$$\text{subject to } \mathbf{s}_{t+1}^{(\mathrm{D})} \sim f_{\mathrm{D}}(\cdot \mid \mathbf{A}_t^{(\mathrm{D})}, \mathbf{A}_t^{(\mathrm{D})}) \quad \forall t \tag{1b}$$

$$\mathbf{s}_{t+1}^{(\mathrm{A})} \sim f_{\mathrm{A}}(\cdot \mid \mathbf{S}_t^{(\mathrm{A})}, \mathbf{A}_t) \quad \forall t \tag{1c}$$

$$\mathbf{o}_{t+1} \sim Z(\cdot \mid \mathbf{S}_{t+1}^{(\mathrm{D})}, \mathbf{A}_t^{(\mathrm{A})}) \quad \forall t \tag{1d}$$

$$\mathbf{a}_t^{(\mathrm{A})} \sim \pi_{\mathrm{A}}(\cdot \mid \mathbf{H}_t^{(\mathrm{A})}), \ \mathbf{a}_t^{(\mathrm{A})} \in \mathcal{A}_{\mathrm{A}}(\mathbf{s}_t) \quad \forall t \tag{1e}$$

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$\mathbb{E}_{(\pi_{\mathrm{D}}, \pi_{\mathrm{A}})}$ denotes the expectation of the random vectors $(\mathbf{S}_t, \mathbf{O}_t, \mathbf{A}_t)_{t \in \{1, \ldots, T\}}$ when following the strategy profile $(\pi_{\mathrm{D}}, \pi_{\mathrm{A}})$.

(1) can be formulated as a zero-sum Partially Observed Stochastic Game with Public Observations (a PO-POSG):

$$\Gamma = \langle \mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (\mathcal{A}_i)_{i \in \mathcal{N}}, (f_i)_{i \in \mathcal{N}}, u, \gamma, (\mathbf{b}_1^{(i)})_{i \in \mathcal{N}}, \mathcal{O}, Z \rangle$$

Existence of a Solution

Theorem

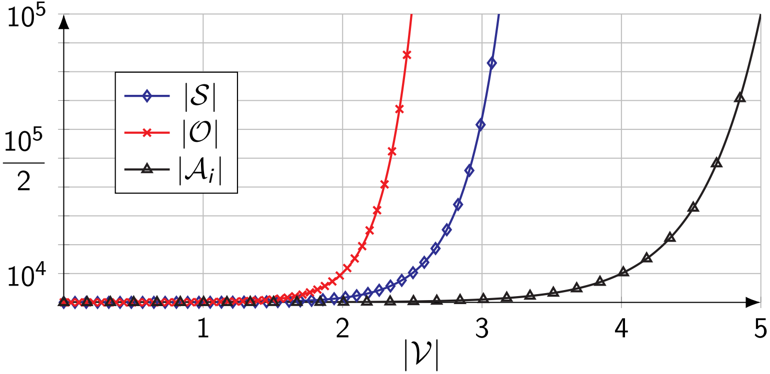
Given the PO-POSG Γ *(2), the following holds:*

(A) Γ *has a mixed Nash equilibrium and a value function* $V^* : \mathcal{B}_{\mathrm{D}} \times \mathcal{B}_{\mathrm{A}} \to \mathbb{R}$ *that maps each possible initial pair of belief states* $(\mathbf{b}_1^{(\mathrm{D})}, \mathbf{b}_1^{(\mathrm{A})})$ *to the expected utility of the defender in the equilibrium.*

(B) *For each strategy pair* $(\pi_{\mathrm{A}}, \pi_{\mathrm{D}}) \in \Pi_{\mathrm{A}} \times \Pi_{\mathrm{D}}$*, the best response sets* $B_{\mathrm{D}}(\pi_{\mathrm{A}})$ *and* $B_{\mathrm{A}}(\pi_{\mathrm{D}})$ *are non-empty and correspond to optimal strategies in two Partially Observed Markov Decision Processes (*POMDP*s):* $\mathscr{M}^{(\mathrm{D})}$ *and* $\mathscr{M}^{(\mathrm{A})}$*. Further, a pair of pure best response strategies* $(\tilde{\pi}_{\mathrm{D}}, \tilde{\pi}_{\mathrm{A}}) \in B_{\mathrm{D}}(\pi_{\mathrm{A}}) \times B_{\mathrm{A}}(\pi_{\mathrm{D}})$ *and a pair of value functions* $(V^*_{\mathrm{D},\pi_{\mathrm{A}}}, V^*_{\mathrm{A},\pi_{\mathrm{D}}})$ *exist.*

The Curse of Dimensionality

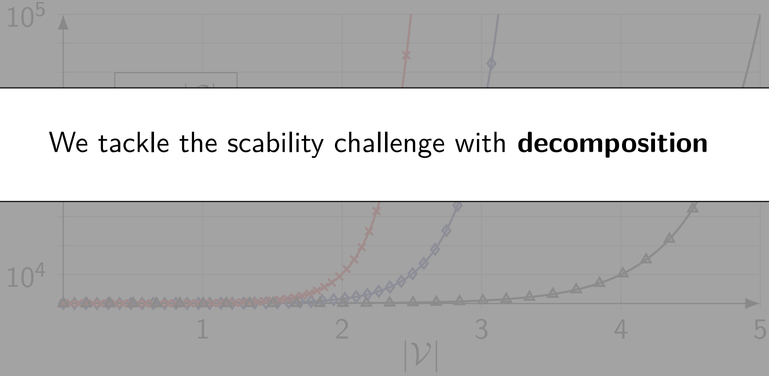
- While Γ has a value, computing it is intractable. The state, action, and observation spaces of the game **grow exponentially** with $|\mathcal{V}|$.



Growth of $|\mathcal{S}|$, $|\mathcal{O}|$, and $|\mathcal{A}_i|$ in function of the number of nodes $|\mathcal{V}|$

The Curse of Dimensionality

- While (1) has a solution (i.e the game Γ has a value (Thm 1)), computing it is intractable since the state, action, and observation spaces of the game **grow exponentially** with $|\mathcal{V}|$.

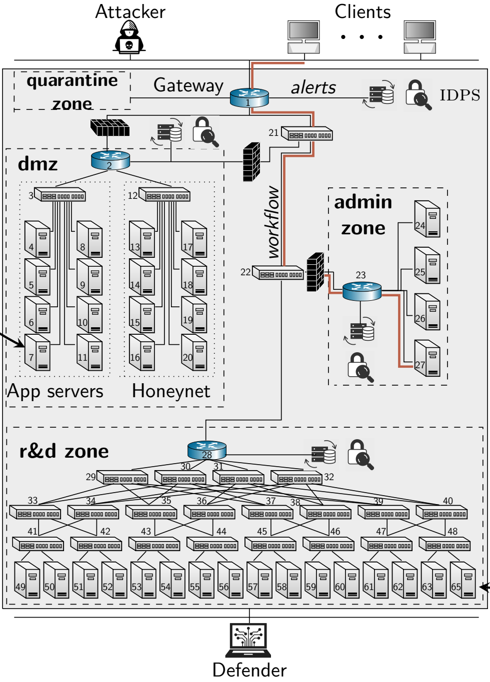


We tackle the scability challenge with **decomposition**

Growth of $|\mathcal{S}|$, $|\mathcal{O}|$, and $|\mathcal{A}_i|$ in function of the number of nodes $|\mathcal{V}|$

Intuitively..

The optimal action here...



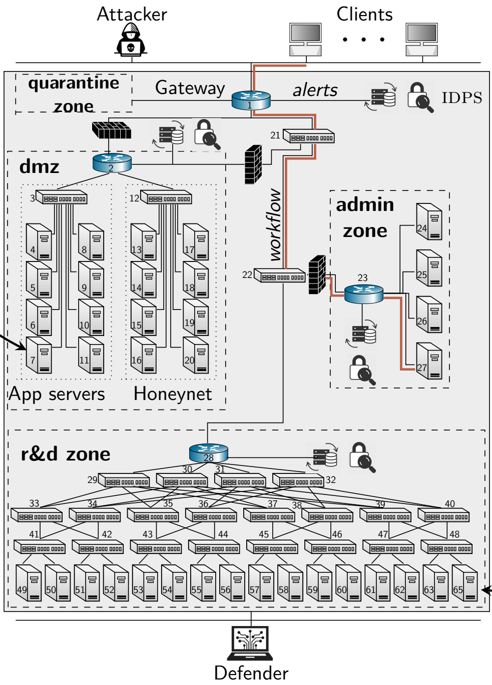
Does not directly depend on the state or action of a node down here

Intuitively..

The optimal action here...

But they are not completely independent either.

How can we exploit this structure?



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Our Approach: System Decomposition

To avoid explicitly enumerating the very large state, observation, and action spaces of Γ, we exploit three structural properties.

1. **Additive structure across workflows.**
   - The game decomposes into additive subgames on the workflow-level, which means that the strategy for each subgame can be optimized independently
2. **Optimal substructure within a workflow.**
   - The subgame for each workflow decomposes into subgames on the node-level that satisfy the *optimal substructure* property
3. **Threshold properties of local defender strategies.**
   - The optimal node-level strategies for the defender exhibit threshold structures, which means that they can be estimated efficiently

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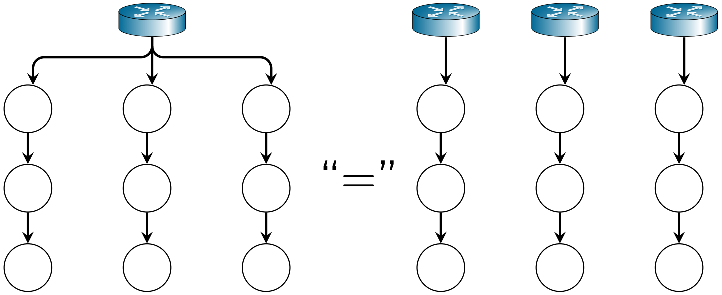
Additive Structure Across Workflows (Intuition)



- If there is no path between i and j in $\mathcal{G}$, then i and j are **independent** in the following sense:
  - Compromising i has no affect on the state of j.
  - Compromising i does not make it harder or easier to compromise j.
  - Compromising i does not affect the service provided by j.
  - Defending i does not affect the state of j.
  - Defending i does not affect the service provided by j.

Additive Structure Across Workflows

Definition (Transition independence)

A set of nodes $\mathcal{Q}$ are transition independent iff the transition probabilities factorize as

$$f(\mathbf{S}_{t+1} \mid \mathbf{S}_t, \mathbf{A}_t) = \prod_{i \in \mathcal{Q}} f(\mathbf{S}_{t+1,i} \mid \mathbf{S}_{t,i}, \mathbf{A}_{t,i})$$

Definition (Utility independence)

A set of nodes $\mathcal{Q}$ are utility independent iff there exists functions $u_1, \ldots, u_{|\mathcal{Q}|}$ such that the utility function u decomposes as

$$u(\mathbf{S}_t, \mathbf{A}_t) = f(u_1(\mathbf{S}_{t,1}, \mathbf{A}_{t,1}), \ldots, u_1(\mathbf{S}_{t,|\mathcal{Q}|}, \mathbf{A}_{t,\mathcal{Q}}))$$

and

$$u_i \leq u_i' \iff f(u_1, \ldots, u_i, \ldots, u_{|\mathcal{Q}|}) \leq f(u_1, \ldots, u_i', \ldots, u_{|\mathcal{Q}|})$$

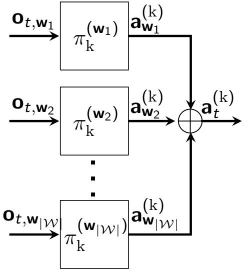
Additive Structure Across Workflows

Theorem (Additive structure across workflows)

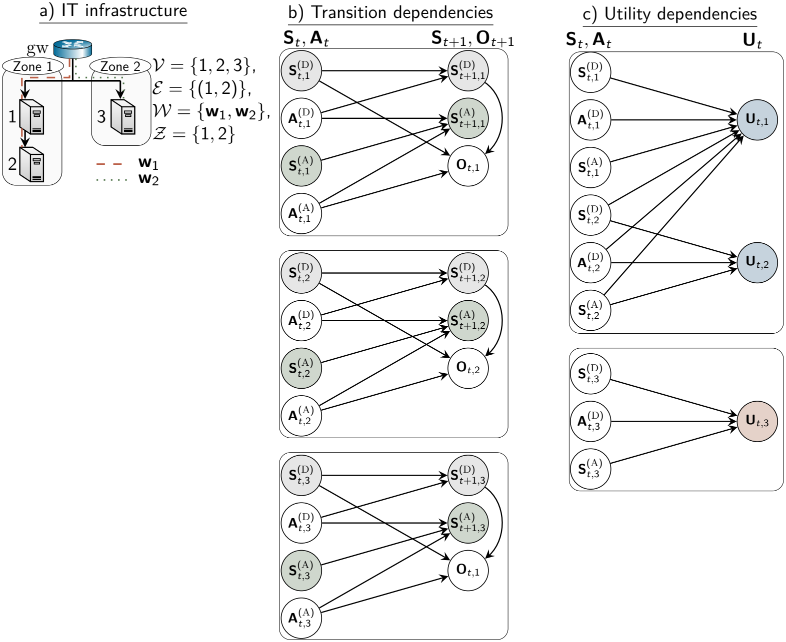
(A) All nodes $\mathcal{V}$ in the game Γ are transition independent.
(B) If there is no path between i and j in the topology graph $\mathcal{G}$, then i and j are utility independent.

Corollary

Γ decomposes into $|\mathcal{W}|$ additive subproblems that can be solved independently and in parallel.



Additive Structure Across Workflows: Example



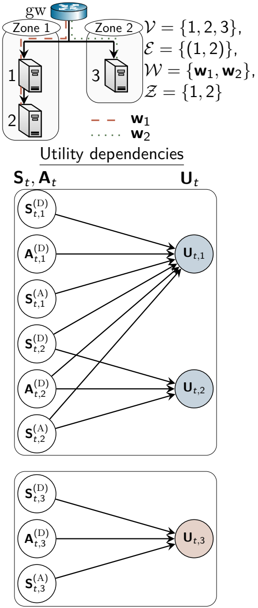
Our Approach: System Decomposition

To avoid explicitly enumerating the very large state, observation, and action spaces of Γ, we exploit three structural properties.

1. **Additive structure across workflows.**
   - The game decomposes into additive subgames on the workflow-level, which means that the strategy for each subgame can be optimized independently
2. **Optimal substructure within a workflow.**
   - The subgame for each workflow decomposes into subgames on the node-level that satisfy the *optimal substructure* property
3. **Threshold properties of local defender strategies.**
   - The optimal node-level strategies for the defender exhibit threshold structures, which means that they can be estimated efficiently

Optimal Substructure Within a Workflow

- Nodes in the same workflow are utility dependent.
- $\implies$ Locally-optimal strategies for each node **can not** simply be added together to obtain an optimal strategy for the workflow.
- However, the locally-optimal strategies satisfy the optimal substructure property.
- $\implies$ there exists an algorithm for constructing an optimal workflow strategy from locally-optimal strategies for each node.



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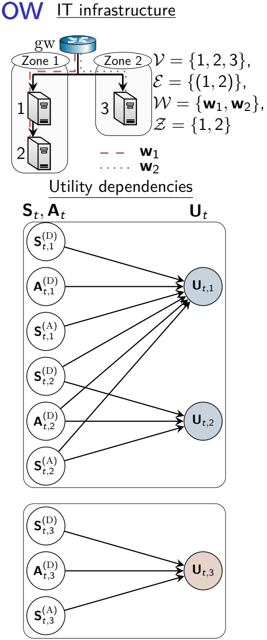
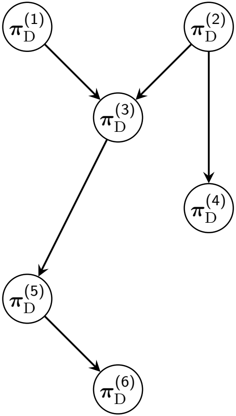


Algorithm for Combining Locally-Optimal Node Strategies into Optimal Workflow Strategies



$(\pi_D^{(i)})_{i \in \mathcal{V}_\mathbf{w}}$: local strategies in the same workflow $\mathbf{w} \in \mathcal{W}$

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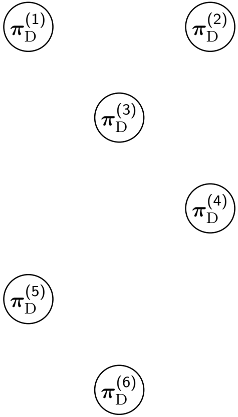

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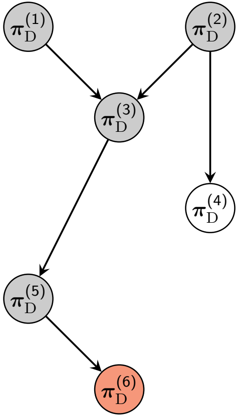
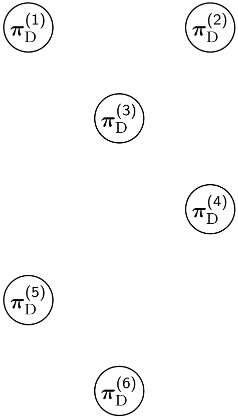


Can redefine the utility function for each node i
to take into account the utility impact on its ancestors.
e.g. utility of node 6 need to include utility impact for $1, 3, 5$.

Algorithm for Combining Locally-Optimal Node Strategies into Optimal Workflow Strategies



Can prove that this utility transformation makes the nodes utility independent.

$\implies$ Optimal substructure.

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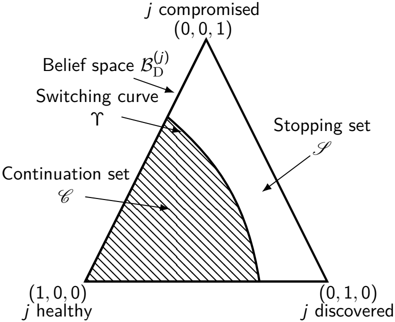
Threshold Properties of Local Defender Strategies.

- The local problem of the defender can be decomposed in the temporal domain as

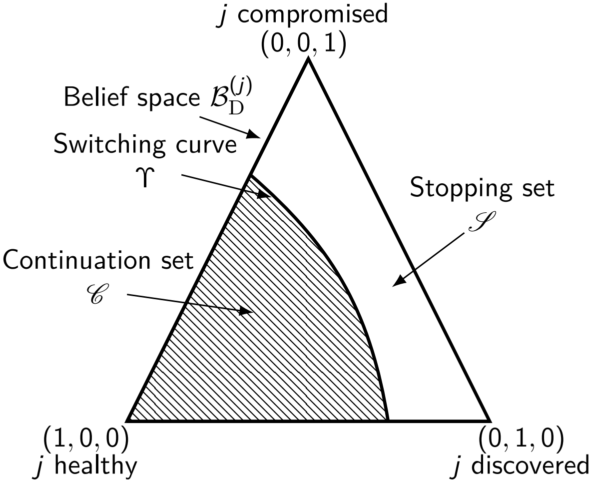
$$\max_{\pi_{\mathrm{D}}} \sum_{t=1}^{T} J = \max_{\pi_{\mathrm{D}}} \sum_{t=1}^{\tau_1} J_1 + \sum_{t=1}^{\tau_2} J_2 + \ldots \quad (2)$$

where $\tau_1, \tau_2, \ldots$ are stopping times.

- $\implies$ (1) selection of defensive actions is simplified; and (2) the optimal stopping times are given by a threshold strategy that can be estimated efficiently:



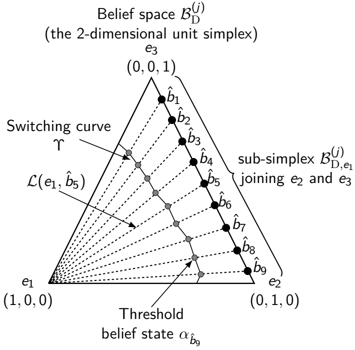
Threshold Properties of Local Defender Strategies.



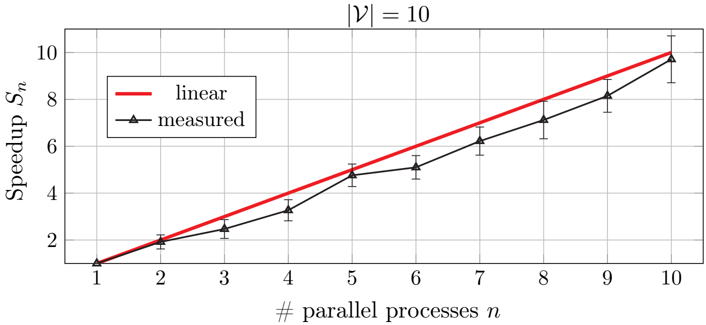
- A node can be in three attack states $s_t^{(\mathrm{A})}$: Healthy, Discovered, Compromised.
- The defender has a belief state $\mathbf{b}_t^{(\mathrm{D})}$

Proof Sketch (Threshold Properties)

- Let $\mathcal{L}(e_1, \hat{b})$ denote the line segment that starts at the belief state $e_1 = (1, 0, 0)$ and ends at $\hat{b}$, where $\hat{b}$ is in the sub-simplex that joins e_2 and e_3.
- All beliefs on $\mathcal{L}(e_1, \hat{b})$ are totally ordered according to the Monotone Likelihood Ratio (MLR) order. $\implies$ a threshold belief state $\alpha_{\hat{b}} \in \mathcal{L}(e_1, \hat{b})$ exists where the optimal strategy switches from C to S.
- Since the entire belief space can be covered by the union of lines $\mathcal{L}(e_1, \hat{b})$, the threshold belief states $\alpha_{\hat{b}_1}, \alpha_{\hat{b}_2}, \ldots$ yield a switching curve Υ.

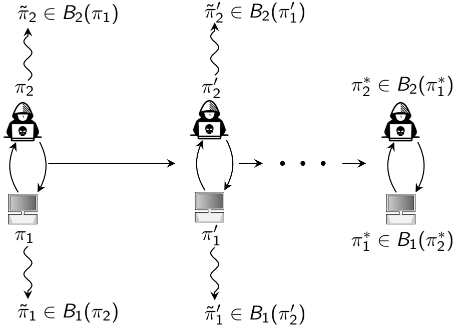


Scalable Learning through Decomposition



Speedup of completion time when computing best response strategies for the decomposed game with $|\mathcal{V}| = 10$ nodes and different number of parallel processes; the subproblems in the decomposition are split evenly across the processes; let T_n denote the completion time when using n processes, the speedup is then calculated as $S_n = \frac{T_1}{T_n}$; the error bars indicate standard deviations from 3 measurements.

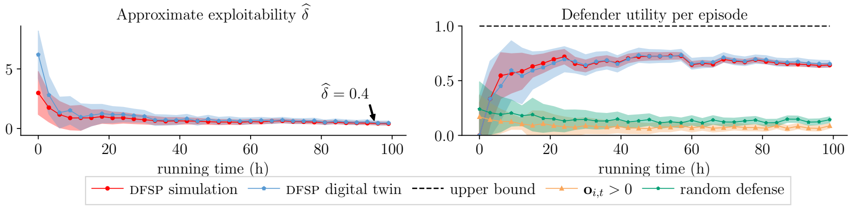
Decompositional Fictitious Play (DFSP)



Fictitious play: iterative averaging of best responses.

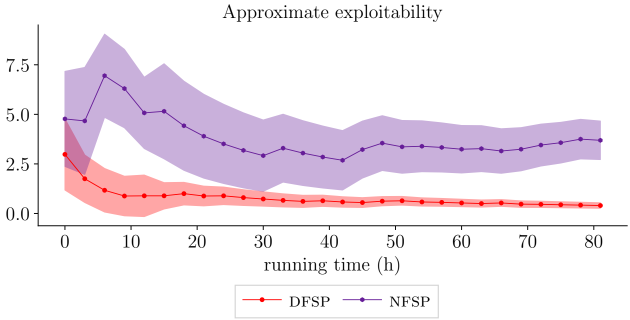
- Learn best response strategies iteratively through the parallel solving of subgames in the decomposition
- Average best responses to approximate the equilibrium

Learning Equilibrium Strategies



Learning curves obtained during training of DFSP to find optimal (equilibrium) strategies in the intrusion response game; red and blue curves relate to DFSP; black, orange and green curves relate to baselines.

Comparison with NFSP



Learning curves obtained during training of DFSP and NFSP to find optimal (equilibrium) strategies in the intrusion response game; the red curve relate to DFSP and the purple curve relate to NFSP; all curves show simulation results.

Conclusions

- We study an **intrusion response use case**.
- We formulate the use case as a **POSG**
- We design a novel decompositional approach to approximate equilibria
- We show that the decomposition allows scalable approximation of equilibria.